Research Paper

Influence of the backfill parameters in distinct element modeling of a backfill masonry arch bridge through the pfc2d software

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Abstract: This paper presents an investigation on how to model a backfill masonry arch bridge using the discrete element method (DEM). The PFC2D software is used for this purpose. A certain backfill masonry arch bridge is modelled, with different types of fill, mortar and voussoir materials. A biaxial test has been programmed using 'FISH' code to obtain the micro-parameters used in the model so that they match the macro-parameters obtained from a real biaxial test. The arch is loaded until its collapse in three different load cases. Further discussion on the influence of the backfill parameters is presented as well as a sensitivity analysis for the arch bridge.

Keywords: Discrete Element Method; Backfill Masonry Bridge; PFC2D; Geomechanical modelling; Soil-structure interaction.

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1. Introduction

Masonry constructions are an essential part of the historical architecture of our towns and cities. Specially in the former, there are still many constructions made of masonry, ashlar or variants. Churches, palaces or castles spread through our land, and represent not only a legacy to preserve, but a fundamental cultural, identity and tourist element. [1].

Within the masonry works, bridges are a very important part. Masonry bridges base their resistant scheme in their arch shape through the concept of antifunicularity (Arenas de Pablo, J.J.) [1], (Huerta Fernandez, S.) [2], (Heyman, J.) [3].

The basic idea of the arch bridge is the arrangement of stone elements called voussoirs with an arch geometry that leads to internal compression forces, exerted on a voussoir against a voussoir, perpendicular to the joints and compressing them uniformly, bringing the loads to the foundation. If the internal pressure line caused by a given state of charge passes through the traced arch, we are facing a stable arch. Further explanation of this phenomena can be found in the works of J. Heyman [3] and Sejourné [4].

The stone materials resist mainly the compression efforts and little or nothing the tractions and the shear forces. In this way, the structural typology of the arch matches perfectly with the working form of the stone.

Masonry bridges with backfill also have the collaboration of the so-called backfill. First of all, it is necessary to clarify what the term backfill refers to, since it is not entirely clear, within the group of the masonry arch bridges. Under this term we want to refer everything located between the factory vaults and the running surface.

Following J.A. Martín-Caro Álamo [5] and Alejandro Ramos Casquero [6] fills are classified from the structural point of view into two groups. On the one hand, rigid or cemented fill materials and on the other loose ones. Rigid filling is usually located in the area near the beginning of the vaults, while the loose one is in the area near the key of the vault.

The structural meaning of the fillings is very important within the overall behavior of the vaults. The loose filling serves to receive the loads that act on the running surface and distributes them along the filling to the back of the vault. Thus, local effects of application of the overloads are reduced, spreading the efforts on a much larger surface. It also helps to center the line of pressures on the thickness of the vault by increasing the weight of the structure itself. The rigid filling has mechanical properties similar to the factory of the vaults, piles and abutments. The effective thickness of the arch, as well as its carrying capacity, are increased thanks to this filling.

Several methods for the analysis of the bearing capacity of the backfill masonry arch bridges are available. Among them we can mention simple conservative methods (MEXE, see UIC Code 778-3R, 1994) [7] and other more recent methods aided by the development of computer calculation methods such as the FEM (finite element method) (Tower and Sako, 1983) [8], (Tower, 1985) [9], (Crisfield, 1984) [10] or the DEM (discrete element method) (A.Tavalingam, 2001) [11].

In the present paper, the DEM will be used, considering that this method offers certain alternative advantages in comparison to the calculation techniques based on continuous methods.

FEM has usually been applied to develop models for masonry vaults, and an interesting example can be found in (Omar Moreno Regan, 2016) [12].

Following (Bolton, 2000) [13] and (González Navarro et al, 2014) [14], we can assert that the calculations, that geotechnical engineers use the most today are elastic calculations, which use an elastic modulus “E”, or plastic analysis that check the equilibrium limit, through a cohesion “c” and a friction angle “Φ”. There is an important problem here in determining these parameters. Soils are not homogeneous materials, but have a great variability. The orientation, the type of soil, the tensional range as well as dilated effects over time, such as consolidation, play a very important role. The micromechanics of particles arises here to solve these difficulties. However, it is important to emphasize that the micromechanics of particles does not arise to replace these elastic and plastic methods, but as a complement to them, to understand the mechanical phenomena that occurs in soils on a smaller scale. In summary, this problem can be presented with the following points:

- (1) Soils are heterogeneous materials, with great variability of their properties depending on a great variety of situations.
- (2) Characterization of soils through classical parameters such as E, c or Φ is complicated, since these parameters are not constants. Although DEM models do not offer a solution to this, they provide a new approach, by using a calculation procedure that updates the contacts between particles in each calculation cycle. Thus, the contact forces between particles are taken into account.
- (3) Certain results through the adjustment of curves are dimensionally incorrect.
- (4) There are some mechanics of soil behavior that are not really understood yet.

2. Discrete element modeling

The Discrete Element Modeling (DEM) is a numerical solution used to describe the mechanical behavior of discrete bodies. A DEM model considers the structure as a set of separate particles, "discrete elements", in which each of them is able to move and deform independently from the other, thus creating new contacts where the efforts are transmitted from one block to another, causing new tensions and deformations in them. Hence, particles can interact with each other, considering their normal and shear stiffnesses, as we can see in Fig.1. The two most extended DEM methods for masonry structures are UDEC and DDA. UDEC (Cundall, 1971, 1988) [15] [16] is a stepwise method in which the masonry blocks and their contacts are deformable. The blocks are divided into simpler forms (2D triangles) that serve as a kind of uniform chain of finite elements. DDA (Shi, 1989) [17] is also a method of step-by-step analysis. In DDA each element has a reference point, forming deformable contacts, considering the system's stiffness matrix as well (UDEC does not consider it). Masonry structures can be represented using DEM as assemblies of particles as seen in (José V. Lemos, 2007) [18] and (Joseph P. Morris, 2009) [19]

Fig 1. should appear here.

3. Methodology

A fundamental aspect in the study of masonry bridges with backfill by using DEM is the interaction between the filling and the masonry. The role that micro-mechanics play in the behavior of the filling is really important.

In this paper, we will use the calculation environment called PFC2D (Particle Flow Code) version 4.0 in two dimensions, developed by the Itasca Consulting Group Inc [21]. This method, also based on the DEM method developed by Cundall, models the movement and interaction between particles, allowing them moving independently and only interacting in the contacts between adjacent particles. The movement of each particle is defined by Newton's second law. The Newtonian force-displacement law is used to define the contact forces in the perpendicular and tangential directions, considering the relative displacements between the particles and the rigidity of the contact in the corresponding directions. These forces are used to calculate the accelerations, speeds and finally the displacements. This calculation cycle is repeated throughout the analysis.

The calculation cycle consists of a repeated application of the law of motion for each particle and of a law of forces-displacements for each contact. Contacts, which can be formed between two balls or between a wall and a ball, are formed and broken automatically during the simulation. At the beginning of each calculation cycle, the set of contacts is updated from the known positions of the balls and contacts and then the force-displacement law is applied to each contact to update the contact forces based on the displacement relative between two entities. Subsequently, the law of motion is applied to each particle to update its speed and position based on the force and moment resulting from the contact forces and the rest of the acting forces in the model. The contact model used is a linear model in which the rigidity of the particles is kept constant over time. The rigidity of the contact is given by the parameters 'kn' and 'ks' that correspond to the normal and transverse stiffness of each element.

It is also possible to create sets of particles by joining several of them. This type of particle groups behaves as if they were rigid blocks, assigning them certain parameters. Also, in addition to the contacts between particles - called 'contact bonds' – it is possible to introduce another type of contact called 'parallel bonds', which allows transmitting both forces and moments in the contacts between particles. The contact bonds can only transmit forces in the contacts. The forces in the contacts between particles that determine the equilibrium of the system are calculated by following the movements of the particles individually.

3.1. Case study

The case analyzed in this paper tries to determine the influence of the filling on the back of an arch bridge in the general behavior of the structure itself. The geometry of the studied bridge is shown in the diagram attached in Fig.2. The inner radius of the bridge used is 3 meters, and the outer radius is 4 meters, with an arch thickness of 1 meter.

Fig 2. should appear here.

A fundamental phase is the construction of the arch. The geometry of the arch is a very special case that requires some attention. In order to study different cases of round arches, with different free lights and vault edges, a worksheet has been created with the idea of helping to model the vault.

The thickness of the arch has been divided in three particles. It could be done with more or less balls if desired. The greater the number of balls, the greater the regularity of the geometry, especially in the back of the vault. Thanks to the worksheet created, from the inner radius of the arch and the thickness of the vault, it is possible to iterate with different particle radii to achieve a perfect arch geometry. To do this, the thickness of the arch is divided into three concentric sectors, as shown in Fig.3. The balls of each sector have the same radius between them. In this way, the balls only touch each other at a point, which is a condition when introducing the particles in PFC2D. The iteration begins with the radius of the balls of the first sector (red balls). From the iteration with this radius the radii of the particles of the other two sectors (green balls and purple balls) are obtained, in such a way that the sum of diameters of the particles of the three concentric sectors coincides with the thickness of particles, as it appears in the following scheme in Fig. 3.

Fig 3. should appear here

Once the radii of particles have been obtained, the number of voussoirs used is determined. This will be important, since we will have to use an entire number of segments.

This is one of the most complicated parts of the arch modeling, since the relationship between radii of particles and thickness of the vault is not obvious or immediate. Perhaps here is one of the weakest points of PFC2D. The introduction of geometry is at least laborious in this type of problems in which geometry is complicated. Therefore, and to facilitate further analysis with more cases, it has been decided to create a spreadsheet to obtain the geometry of the model.

Thus, a 'macro' has been created that greatly eases the introduction of geometry. As explained, the created spreadsheet allows to obtain the radius of the particles to be introduced to model the vault of the arch. In turn, another sheet has been created to obtain the X and Y coordinates of all the particles, and a 'macro' that allows exporting the coordinates of all the particles to a .txt format that can be executed in PFC2D. In this way, no code is written directly into the program (something extremely cumbersome), but from the parameters of the arch geometry (radius and thickness of the vault) it is possible to create a .txt file containing the complete geometry of the model in a very short time, almost automatically (just iterating with the radius and executing the created macro).

Fig 4. should appear here.

The supports of the arches, as well as the lateral limits of the analyzed bridge, are created as 'wall' elements. These walls serve as limits of the created model, to support the arch and retain the filling. It should be noted that up to this point, mortar infill joints between voussoirs have not been created, but that the arch acts as a set with particles strongly bound by 'parallel bonds'.

In the process of building an arch of this type, the arch is first created, with the different voussoirs, and once it is in equilibrium, the structural fill in the backfill begins to be placed.

Once the vault geometry is created, the fill is introduced. To do this, and to control the porosity of the structural filling, its particles are introduced in several layers, allowing each layer to settle under the action of gravity as shown in Fig. 5. In this case, a loop has been programmed in which the different filling layers are introduced until the geometry of the model is completed. The contact is also assigned 'contact bonds'. Finally, the joints between voussoirs are created, with their particular properties. The joint planes can be defined in PFC2D to represent surfaces where slip and separation between joined groups of particles is possible. These properties are such that they allow one segment to slide over another as long as the friction force between them is overcome. This frictional force is a parameter that is introduced into the program.

Subsequently, the arch is left again to adjust to its new equilibrium position, working together with the filling. Thus, the calculation model is complete, ready to analyze under the action of different actions. In Fig.5. the modeling of the bridge obtained is shown. It shows the different voussoirs obtained after placing the joints. The compressive forces between the particles that form the structural arch are also drawn in black.

Fig 5. should appear here.

Tables 1 and 2 show the properties of fillings, masonry voussoirs and walls used in the simulation. Parameters "kn" and "ks" correspond to the normal and transversal rigidities. "Density" indicates the specific weight. Parameters "n_bond" and "s_bond" indicate the perpendicular and transverse forces in the "contact bond" contact. "pb_kn" and "pb_ks" indicate the normal and transverse stiffness in the "parallel bond" contact. Units are indicated in the table.

Table 1. should appear here.

Table 2. should appear here.

In the present paper, the influence of the type of filling is analyzed with two models, one for a medium soil and another for a soft soil.

The value of the properties to be introduced will depend on the material of the case analyzed. Here lies another difficulty in the use of the program. Choosing the properties that represent the actual material from the properties deduced from laboratory tests is at least laborious. While in continuous codes these values are usually introduced directly, in PFC2D we go to the most basic level, which synthesizes the behavior of the material from the micro-components or equivalent grains that make up the material. The process used to reach these properties is as follows: First certain initial properties are assumed; then, tests are performed on available samples of that material. Subsequently, the results are compared with the desired response in certain tests. If there is correspondence in those tests, those properties will be used in the complete analysis. Otherwise it will iterate again. In order to calibrate the parameters of the contact junctions, the friction, as well as the rigidities of the contacts, a biaxial test has been programmed in which the material with the same properties that will be used later in the complete model is evaluated, so that an appropriate relationship can be reached between the micro and macro properties of the material.

Loads are applied through a 'loading wall' which is given a fixed speed in the vertical direction, which is also directly related to the value of the action to be applied. Perhaps here is another weak point of the software, as loads cannot be applied directly to walls. The collapse load is obtained by monitoring the resulting force in the 'loading wall'. In this simulation, the loading wall is applied in different positions as shown in Fig.2. In this way, three possible positions of the load are analyzed: in the center of vain (CASE 1), 75 cm to its left (CASE 2), and to another 75 cm to the left (CASE 3).

4. Results

The failure load obtained is 420 kN in Case 3 and medium soil, due to the formation of four hinges, according to ultimate loads. Once Case 3 is established as the worst load case, several simulations have been carried out in order to obtain more information on how the backfill influences the load capacity of the bridge. The following conclusions can be drawn with respect to the influence of the type of filling on the general behavior of the masonry bridge, as well as other sensitivity studies:

DEM PARAMETERS SENSITIVITY

- On the one hand, the perpendicular force in the n-bond contact is the determining parameter when obtaining the breaking load. The s-bond transverse force has very little influence since the predominant type of fracture is a mechanism in which no slippage occurs. However, the value of "s_bond" cannot be below a minimum since this parameter prevents a shear break mechanism from occurring, and is needed to give the initial stability to the arch.
- In order to analyse the influence of different types of soils and materials, the friction angle of the mortar varied from values from 30º to 40º, as well as the friction angle of the fill, which varied from 35º to 45º. The results suggest that the collapse load is not significantly influenced by the change in this parameter. The friction coefficient "k" is related to the friction angle by $ø=\tan^{-1} k$.

Fig 6. should appear here.

LOAD POSITION

- The critical position for the load is 1/6 of its span when the arch is analyzed alone. Using the real arch, with the filling, it can be inferred that the position of the critical load moves to 1/4 of its span. This agrees with the theoretical predictions for a semicircular arch like the one studied. (Fairfield CA., 1994) [22].

Fig 7. Should appear here.

BACKFILL PARAMETERS

- The use of medium sand as a filling causes smaller deflections than in the case where loose sand is used. Therefore, the rigidity of the filling has a significant importance in the displacements, causing larger ones in cases where there are less rigid fillings.

Fig 8. should appear here.

- The distribution of forces in the filling is substantially different. The medium sand (more rigid) distributes the load in a wider region, with which the tensions produced in the arch are smaller. Again, the importance of the stiffness of the filling is fundamental for the overall behavior of the bridge.
- A compact filling, with high elastic limit value reduces tensions in the vault. The tensions in the filling increase but not enough to exceed its stress limit.
- The fill height also influences the collapse load. The results show clearly that the failure load increases as the fill height does.

Fig 9. should appear here

5. Conclusions

As final considerations with respect to the DEM method to analyze masonry bridges with backfill fillings, it can be established that the DEM and FEM analysis techniques involve a substantially greater modeling effort with respect to the approximate techniques such as the MEXE or limit analysis as the method of the mechanism.

However, if the analyzed case requires a more sophisticated analysis due to variations in the use of the bridge, or there are important pathologies, or simply do not reach an acceptable failure mode with the existing simplified methods, the use of DEM analysis has full force.

The really interesting thing would be to use several types of analysis. A first basic study could be done when we are facing structures that are in good condition, with normal geometries and under stress. This first level of analysis could be carried out with the aforementioned simplified methods (MEXE, limit analysis). However, in the case where we are facing a singular structure, DEM analysis should be used. This type of studies requires, as it has been seen, field work, sample collection and laboratory analysis, since the choice of the mechanical properties of the materials conditions the correct response of the calculation model used.

After carrying out the present modeling using the DEM method, the following advantages, disadvantages and limitations of this method of analysis are identified in relation to other frequently used methods.

On the one hand, finite element methods (FEM) have difficulties to correctly model the segments and joints using meshes. When large displacements occur in the interfaces it is not possible to create a new mesh, and in this way, update the existing contacts or create new ones. Likewise, the introduction of the parameters that form the constitutive law of the material must be introduced by the user.

In DEM models, large displacements are allowed and it is possible to model the blocks independently, without the need of joining points as in FEM, in addition to the advantages mentioned above. However, code and modeling are laborious, and computing is expensive in terms of time and work when it comes to modeling large structures.

In this way, it can be concluded that when a detailed study is required in a small model, the DEM method would be the most appropriate and precise method that currently exists. However, as described, modeling and computing is quite complicated. On the contrary, when we are dealing with a general analysis of a large model, the FEM method will be the most appropriate, since it does not require such precision and the modeling is much simpler, although information is lost in the analysis, such as the study of failure mechanisms.

Future research in this field should be aimed at the systematization of this type of calculation and the implementation of the DEM method in simpler and more accessible calculation programs. In this way, precision in the analysis of this type of important structures would be gained, correctly predicting the mechanisms of failure and the interaction between the filling and the structure of the arch.

6. References

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TABLES

				CONTACT BONDS	PARALLEL BONDS	REAL PROPERTIES
	kn & ks (N/m)	density (kg/m3)	Friction coeff	n_bond & s_bond (N/m)	pb_kn & pb_ks (N/m3)	E (MPa) Young's modulus
Filling	1e8	2000	0.6	1e8	-	52
Masonry	1e10	2200	0.6	1e10	1e20	-
Walls	1e20	-	0.3	-	-	-

Table 1. DEM properties for a medium soil.

				CONTACT BONDS	PARALLEL BONDS	REAL PROPERTIES
	kn & ks (N/m)	density (kg/m3)	Friction coeff	n_bond & s_bond (N/m)	pb_kn & pb_ks (N/m3)	E (MPa) Young's modulus
Filling	1e5	1800	0.4	-	-	20
Masonry	1e10	2200	0.6	1e10	1e20	-
Walls	1e20	-	0.3	-	-	-

Table 2. DEM properties for a soft soil.

FIGURES

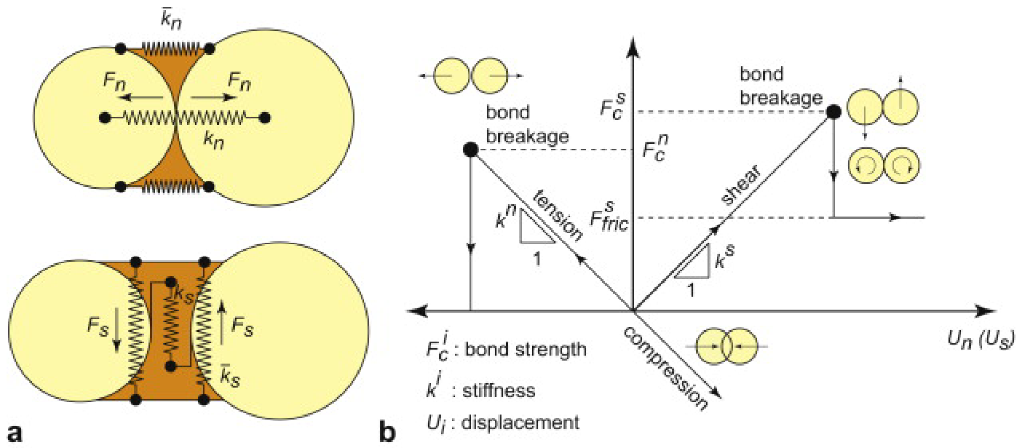

Fig. 1. The parallel bond model implemented in PFC. (a) Normal and shear stiffnesses between particles. (b) Constitutive behavior in shear and tension. Figures redrawn after Potyondy and Cundall (2004) [20].

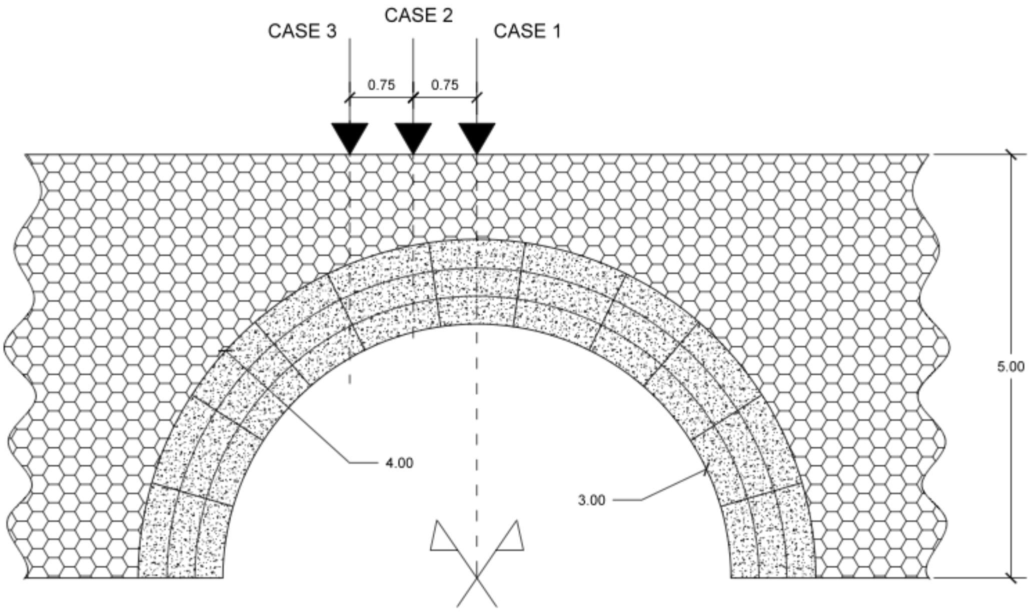

Fig. 2. Case study. The arch is divided into 11 equal segments. The inner radius is 3 meters and the outer radius is 4 meters. The filling height considered is 5 meters. The 3 load cases analyzed are shown.

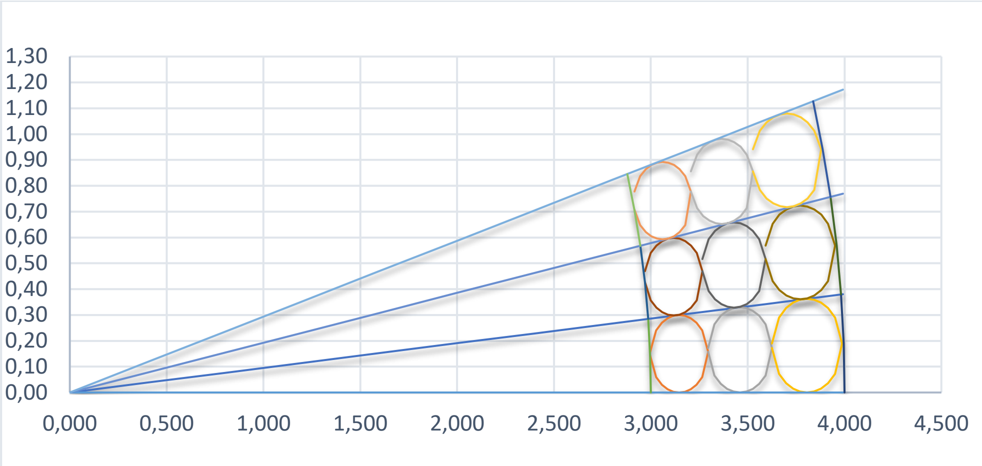

Fig.3. Scheme of division of the vault edge.

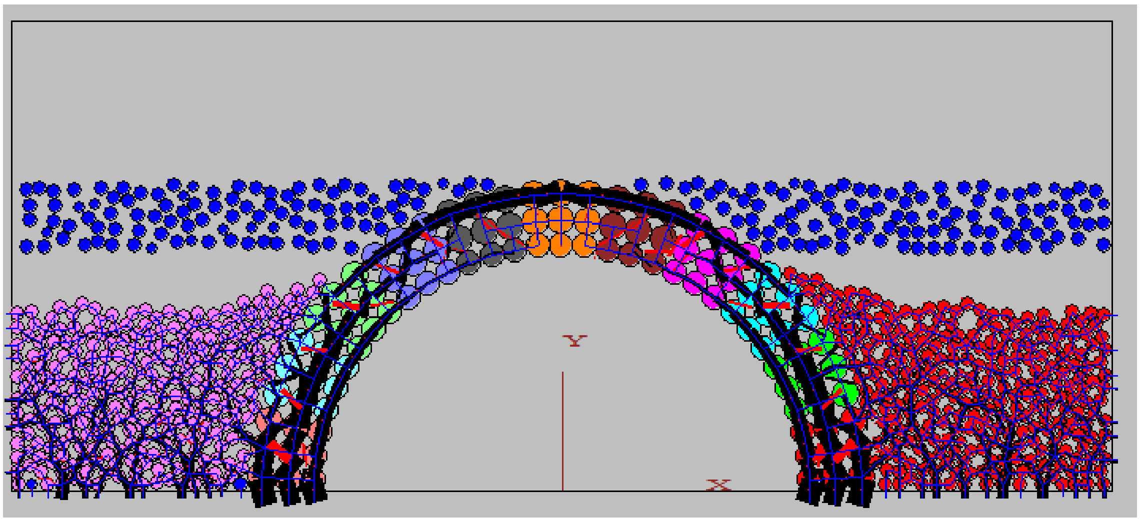

Fig.4. Introduction of the fill by several layers, allowing each one to be settled under the action of gravity. Compression forces between particles are painted in black. It is clear that compression forces pass through the arch.

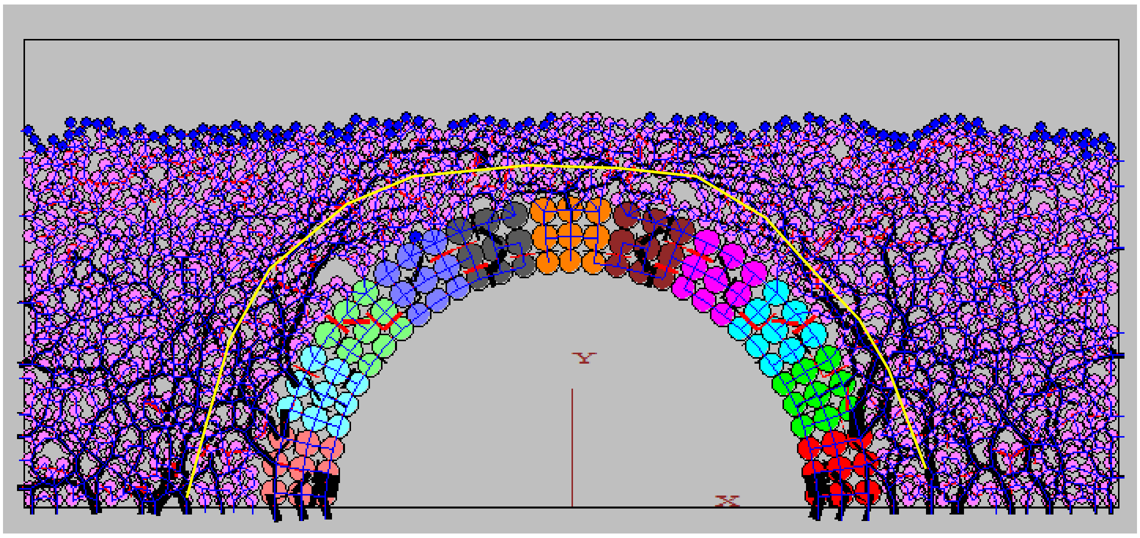

Fig.5. Complete modeling of the bridge. The voussoirs of the arch are appreciated in different colors. In black the compression forces between particles creating a discharge arch through the filling, remarked with the yellow line.

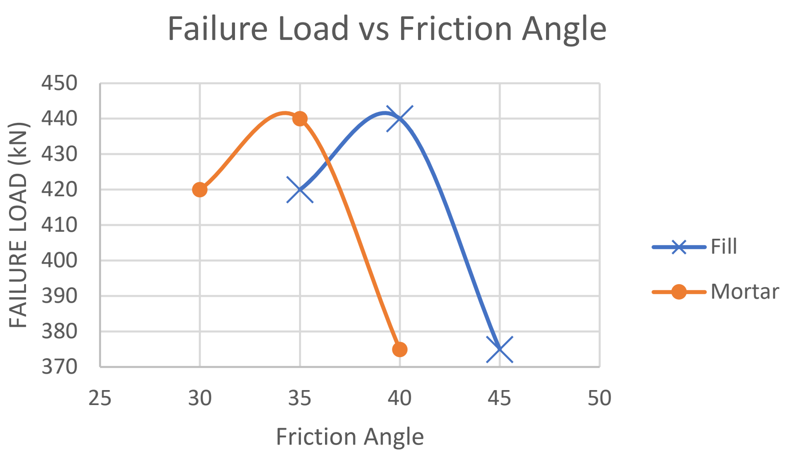

Fig 6. Friction Angle vs Failure Load.

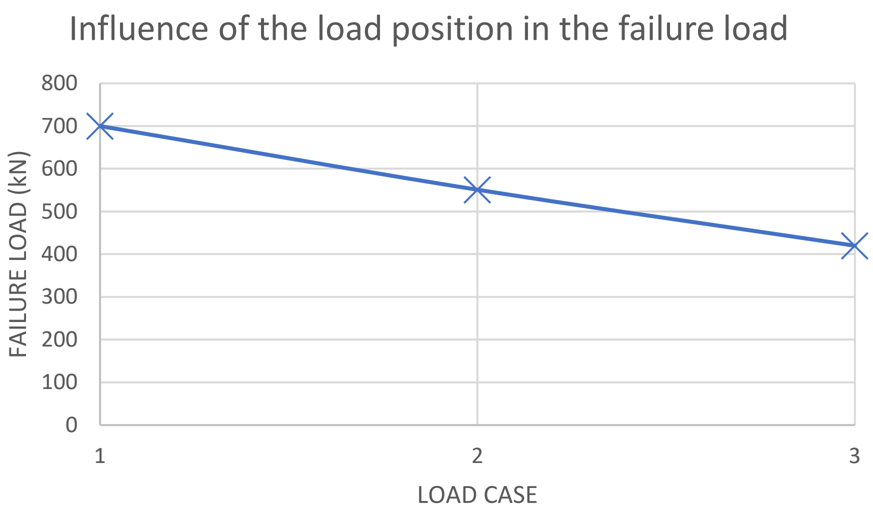

Fig 7. Load Position vs Failure Load.

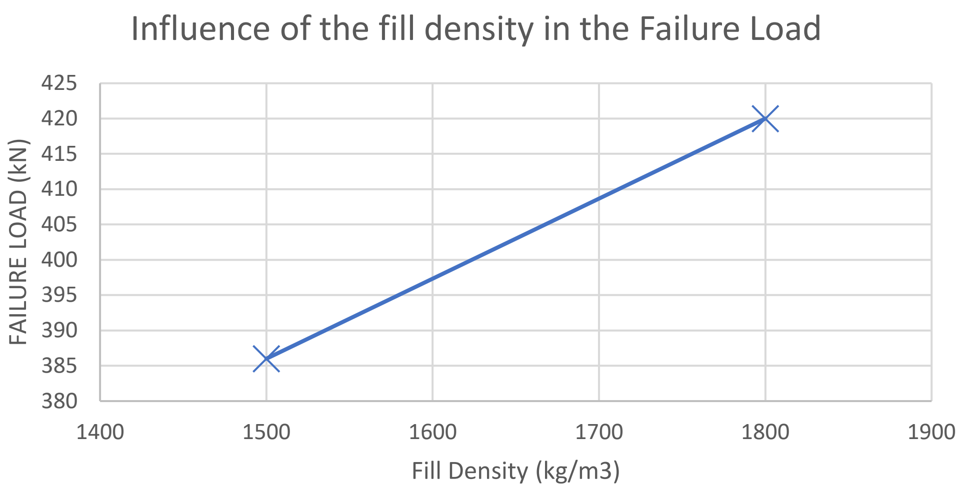

Fig 8. Fill Density vs Failure Load.

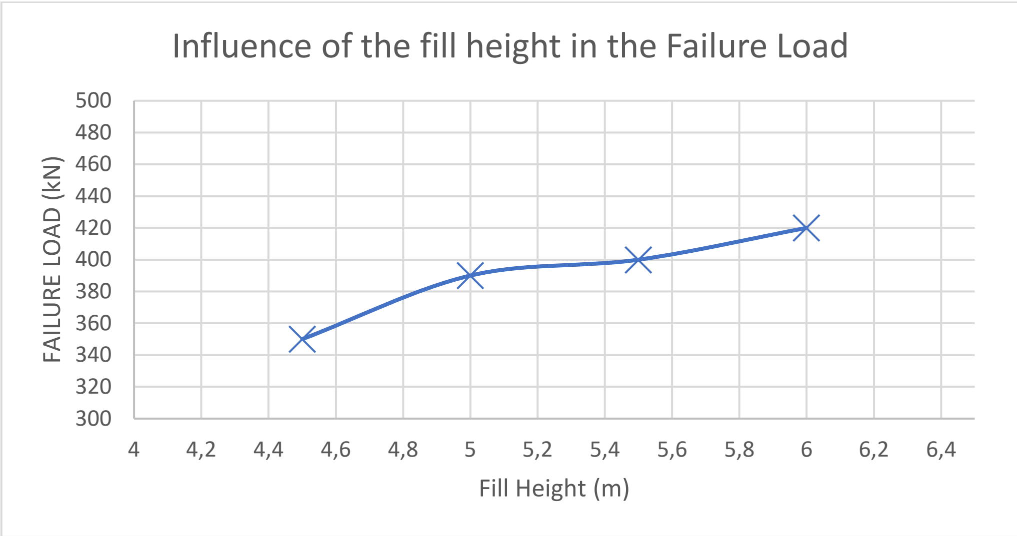

Fig 9. Fill Height vs Failure Load.