COMPUTER AIDS FOR THE INDEPENDENT STUDY OF PHYSICS SUBJECTS IN THE FRESHMAN YEAR OF ENGINEERING STUDIES

N.A. Cordero, J.H. Lucio, P.A. Marcos, L.R. Rodríguez, A. Serna, and R. Valdés

Physics Department. University of Burgos (Spain)

ncordero@ubu.es

Abstract

One of the main challenges in the process of adapting existing syllabi to the European Higher Education Area is the decrease in the number of credits assigned to each subject. This decrease is balanced by the fact that former credits were calculated exclusively by taking into account face-to-face interaction between teacher and student while the new European Credit Transfer System concept of credit encompasses not only this aspect but also the time the student has to spend independently in order to acquire the competences associated with the subject.

In the case of freshman year physics subjects, the student has to grasp many new concepts. Using a traditional textbook is usually not good enough to go beyond the 'forest of formulas' and understand the underlying basic principles in a relatively short period of time. In order to help the student in this task, we have developed several Mathematica notebooks that can be used in his or her independent study time. Mathematica (www.wolfram.com) is a powerful high-level programming language especially suited for analyzing any complex mathematical problem and creating graphics and animations that are an invaluable aid in understanding the meaning of the mathematical expressions that describe the behaviour of a physical system.

We have used Mathematica in our classrooms for nearly 15 years to help students visualize physical magnitudes but until a short time ago it was not possible to effectively use this code for independent study. The situation has changed because the program publisher has made available for free a piece of code called MathPlayer that allows to run Mathematica notebooks. This code in conjunction with the interactive capabilities of Mathematica 7.0 allows any student with a computer to change the input values for a physical system and create graphics and animations that display the behaviour of the different properties of the system.

We have written notebooks devoted to several aspects covered in Physics 101 and 102 syllabi, namely, vector calculus, kinematics and dynamics of particles and systems, fluid mechanics, electromagnetism, and thermodynamics. In this report we present one of these notebooks devoted to the detailed analysis of a damped harmonic oscillator.

Keywords: Physics, mechanics, computer simulation, independent study, Mathematica.

1 BACKGROUND

Let's consider a viscous force $F_v = - R\, x'(t)$ acting on a body of mass m attached to a spring with an elastic constant k. This system is called a damped harmonic oscillator [1]. The force of the spring on the body is given by Hooke's law $F_e = - k\, x(t)$ and the total force acting on the system is $F = F_e + F_v$. Therefore Newton's second law reads

$$m\, x''(t) = - k\, x(t) - R\, x'(t) \;.$$

The general solution for this equation is

$$x(t) = \frac{e^{-\frac{R t}{2 m}} \left[x_0 \sqrt{-4 k m + R^2} \,\mathrm{Cosh}\left(\frac{\sqrt{-4 k m + R^2}\; t}{2 m} \right) + \mathrm{Sinh}\left(\frac{\sqrt{-4 k m + R^2}\; t}{2 m} \right) (x_0 R + 2 m v_0) \right]}{\sqrt{-4 \quad + R^2}} \;.$$

The behaviour of the system is easily characterized in terms of the so called damping ratio

$$\zeta = \frac{R}{2 \sqrt{m k}}$$

If $\zeta = 0$, there is no damping and the system becomes a simple harmonic oscillator.
If $0 < \zeta < 1$, the system is underdamped and oscillates with decreasing amplitude.
If $\zeta = 1$, the system is critically damped and the system returns to equilibrium as quickly as possible.
If $\zeta > 1$, the system is overerdamped and returns to equilibrium with an exponential decay.

2 NOTEBOOK

We have used Mathematica 7.0 [2] to write an interactive notebook that can be read using the free application MathPlayer [3].

The analysis of the movement of the system based on the damping ratio is deceivingly easy. The general solution given in the previous section shows that the behaviour of the system depends in fact on 5 parameters:

- The initial position of the body (x_0)
- The initial velocity of the body (v_0).
- The mass of the body (m).
- The elastic constant of the spring (k).
- The damping coefficient (R).

The notebook we have developed allows the students to understand on their own the behaviour of the system by changing the value of these 5 parameters. Fig. 1 shows the full range of possibilities the application offers. The left part includes input boxes for the five parameters plus a sixth one for the maximum evolution time of the system. This last input is necessary in order to accommodate for the different typical decaying times. The last column shows three outputs. From top to bottom, the damping ratio of the system, a graph showing the evolution of the position of the body as a function of time, and a box displaying the damping regime.

The students can change the value of any of the inputs and see immediately the new behaviour of the system. They can also generate an animation displaying the evolution as one parameter changes.

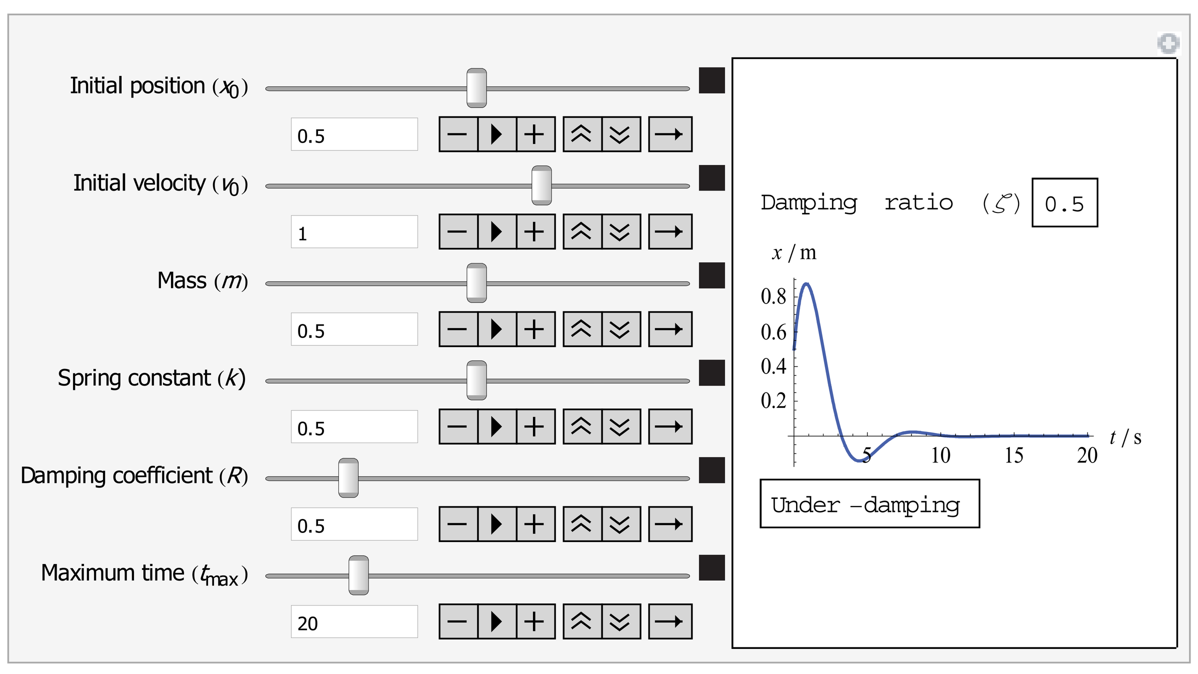

Fig. 1 The notebook with all the menus expanded.

In most cases it is not necessary to give the input values in a numerical way. The students can change them by simply using the corresponding sliders.

We present in the following figures several values for the inputs that lead to interesting behaviours. Fig. 2 depicts the case corresponding to $R = 0$, i.e. no damping, and shows the motion of the body is just a simple harmonic oscillation.

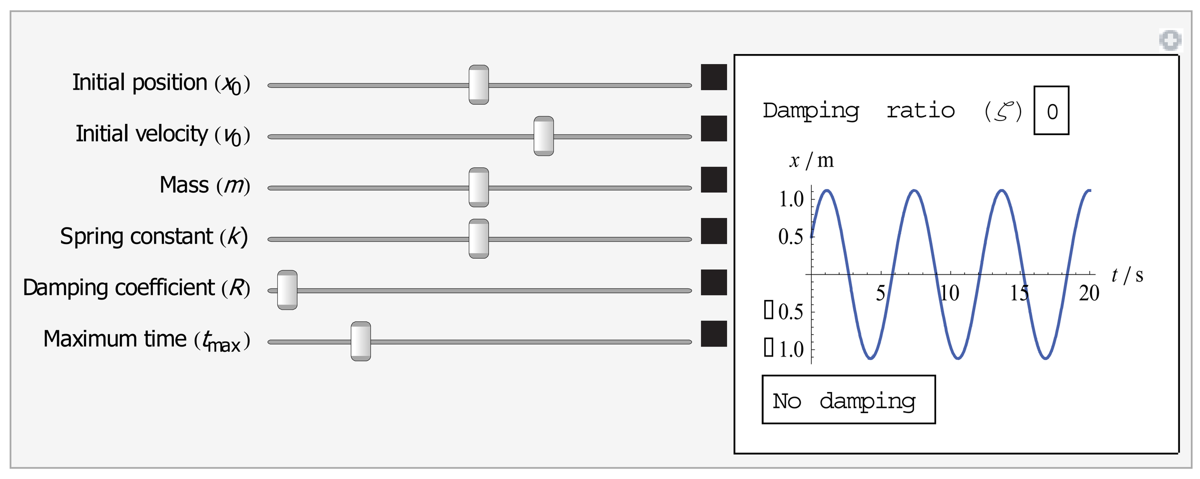

Fig. 2 The notebook with all the menus collapsed and showing an undamped oscillation.

We present in Fig. 3 the situation corresponding to a slightly damped oscillation. It is noticeable the decrease in the oscillation amplitude as the system evolves in time.

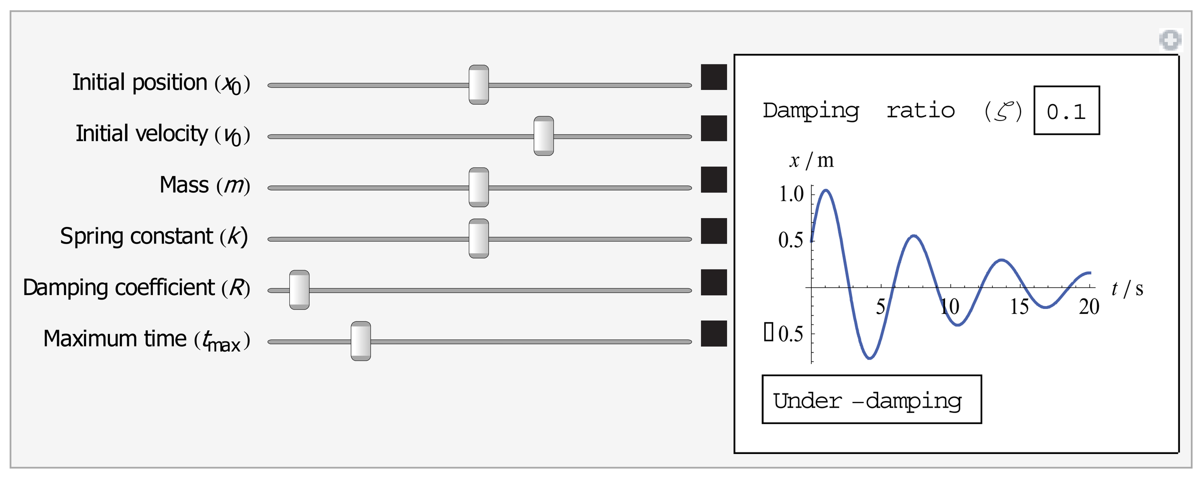

Fig. 3 The notebook with all the menus collapsed and showing an underdamped oscillation

Fig. 4 corresponds to the critical damping situation. In this case, the system tends to go back to the equilibrium configuration as fast as possible and the general solution reduces to a much simpler form:

$$x(t) = \frac{e^{-\frac{k t}{\sqrt{k m}}} \left[x_0 \left(m + \sqrt{k m}\; t \right) + m\, t\, v_0 \right]}{m} \;.$$

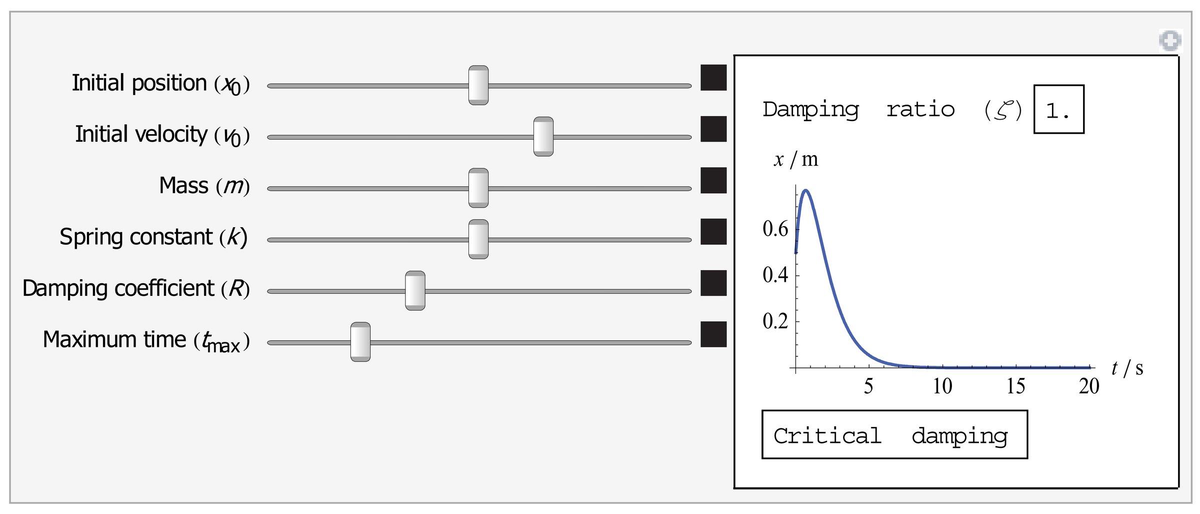

Fig. 4 The notebook with all the menus collapsed and showing a critically damped oscillation

Finally, in Fig. 5 we present and overdamped situation. It is similar to the critically damped one but the system takes longer to reach the equilibrium configuration.

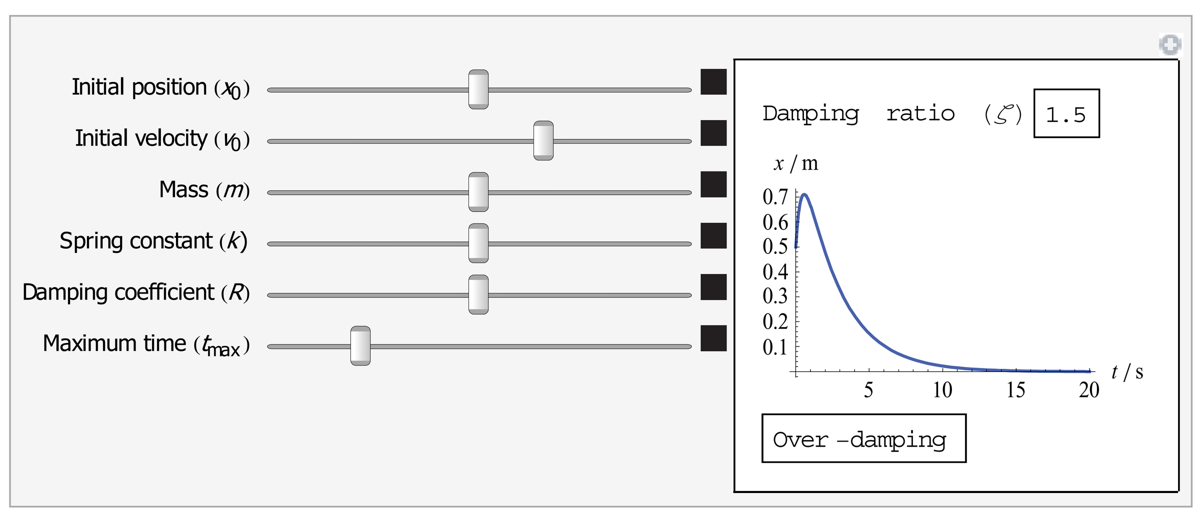

Fig. 5 The notebook with all the menus collapsed and showing an overdamped oscillation

3 CONCLUSIONS

The notebook we have developed is useful for students during their independent study time. They need neither the physical presence of an instructor to understand the changes in the way the system behaves nor buying an expensive code to run the application. We plan to implement its use in our subjects in the fall semester 2010 in order to get feedback from the students and improve it.

REFERENCES

[1] http://en.wikipedia.org/wiki/Harmonic_oscillator#Damped_harmonic_oscillator

[2] http://www.wolfram.com/products/mathematica/index.html

[3] http://www.wolfram.com/products/player/