TITLE:

The shallow arch: a step towards bridges styling in the early 19th century

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ABSTRACT

Several formulations for masonry arches were appearing since 18th century, mostly based on the acquired experience in bridge construction, and that lead to a generalized use of shallow arch. These innovations supposed an evolution in the shape of masonry arch bridges, at the expense of increasing the thickness of voussoirs and vault. The question is: were the mentioned changes related with a clear structural improvement? Or were they mainly aesthetic or fashion-driven changes? This paper tries to give deeper insight and response this query by making a comparison among different formulations, and analyzing the rise to span ratio and backfill influences in both ultimate load capacity and maximum stress. Moreover, a multi-span arch bridge is analyzed, performing a comparison among different typological possibilities, and using examples of real structures in the Carrion river basin (Spain).

KEYWORDS: masonry arch bridge, shallow arch, rise to span ratio, ultimate load, backfill

HIGHLIGHTS

- The arch bridges shape evolution is analyzed in function of the rise to span ratio
- Some influent variables to maximum stress in a masonry bridge are analyzed
- The influence of compacted soil backfill at spandrels is studied
- The variation of ultimate load in a multi-span arch is analyzed using different designs

1. INTRODUCTION

The arch is one of the basic elements in masonry bridge construction. Several authors have analyzed the arch possibilities, such as Alberti and the *Re Aedificatoria* treatise in 1542, and other focused in national cases, like Santiago Huerta [1] as the most outstanding for those built in Spain. Due to its importance, this building technique has been revisited in multiple technical studies analyzing its shape, the barrel thickness, the span, the rise, and the connection among them. These have revealed certain typological predominance during particular historical periods, and even some trend variations. Similarly, these variations have allowed some contributions about the optimal structural behavior, and even about the knowledge of that behavior in different moments.

The knowledge evolution has affected to the shape of some bridge parts: piers, cutwaters, arches… However, this paper is focused only in circular masonry arches. From this point of view, some changes in masonry arch geometry can be observed throughout centuries. In fact, there was a progressive increase of radius of curvature, and hence an evolutionary change of arch shape can be identified. It was a stepped approach towards a segmental arch, searching for more aesthetic types. Main parts of a masonry arch are showed in Figure 1, including the most important magnitudes involved in their behavior. We will use the rise to span ratio (r/s) as a basis of our study, a parameter that can be used to distinguish between different circular arch typologies. Following formula can be derived as mathematical relations:

$$r = \frac{1 + 4 \times (r/s)^2}{8 \times (r/s)} \times s \qquad \theta = 4 \times \left(\frac{\pi}{2} - \arctan \left(\frac{0{,}5}{(r/s)} \right) \right)$$

In function of previous expressions, three main circular arch types can be described as shown in Figures 2, 3, and Table 1: round, semi-shallow, and shallow arches. The last two are subtypes of the generic segmental type.

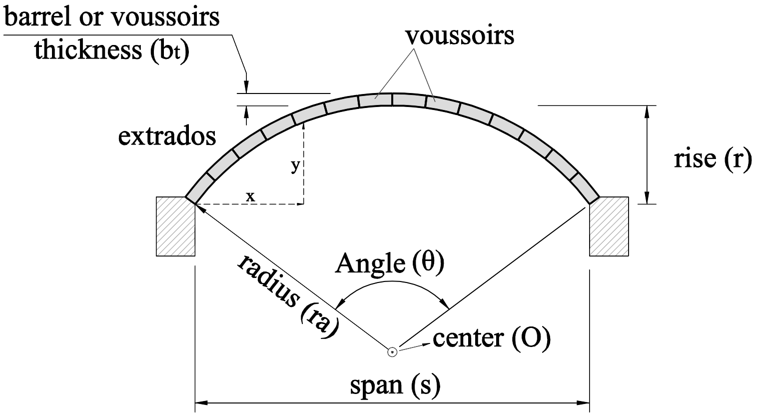

Figure 1. A sketch of a masonry arch, including most important parts and abbreviations used in this paper.

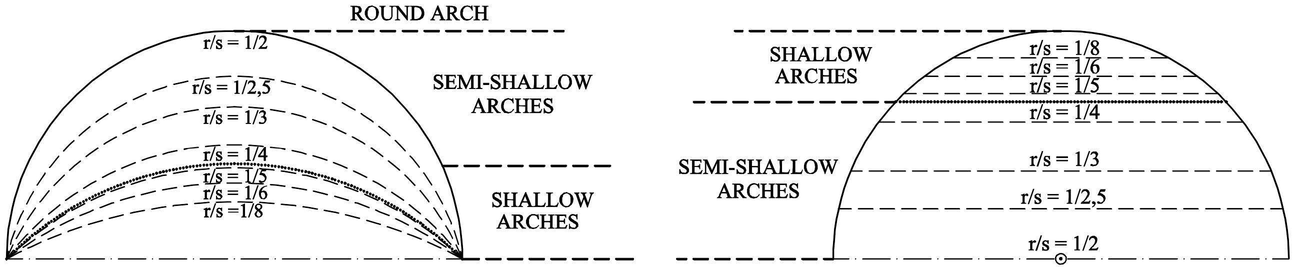

Figure 2. Circular arch types, in function of r/s parameter, for constant span (left) or constant radius (right).

The ultimate load capacity of a masonry bridge is known to be seriously influenced by varying the r/s parameter toward a higher curvature. González Parejo [2] studied different geometries changing θ from 180 to 130 degrees. That was equivalent to change r/s from 1/2 to 1/3.14. In our work, we have increased the analysis up to r/s=1/20 so that we completed previous range. Our structural analysis have included the influence of compacted soil backfill in the ultimate load behavior, which has been shown as a key point by previously published works such as Callaway et al [3], Bjurström y Lasell [4], Cavichi y Gambarota [5], or González Parejo [2]. Moreover, the authors of current paper can assure the use of these backfills in most of the masonry bridges on whose maintenance have been involved. To achieve this, up to twelve real masonry bridges have been analyzed. The paper is completed with a comparative study among different empirical approaches for a real bridge example in the Carrion river basin (Spain), to find an explanation to the evolution of arch shape.

ARCH TYPE	θ	r/s
Round arch	**180º**	**1/2**
Semi-shallow arch	**90º-180º**	**1/2-1/4.82**
Shallow arch	**< 90º**	**< 1/4.82**

Table 1. The table shows three types of circular arch classification in function of r/s parameter.

Figure 3. Three examples in the Carrion river basin (Spain). Upper left (a) Husillos Bridge (endings of 15th century), a round arch; Upper right (b) Villoldo Bridge (endings of 18th century), a semi-shallow arch; below (c) San Isidro Bridge in Dueñas (early 20th century), a shallow arch.

2. THE EVOLUTION OF THE r/s PARAMETER IN THE CARRION RIVER BASIN BRIDGES

Roman bridges have been used, in general, as example for further masonry arches by simple imitation, improving defects and making use of their virtues. Regarding to segmental arches there are some remaining examples in Spain such as the Alconetar Bridge (Cáceres), a remarkable model cited by González Parejo [2]. The study of roman circular arches has shown some connections between barrel thickness and span length, but nothing special about the r/s that could help during a classification. Besides, a few medieval examples of segmental arches are available due to the extensive use of round arch during Middle Ages.

Many authors have analyzed the geometric relations between voussoirs or barrel thickness and span length, the historical recommendations about arch type use, or the connections between previous parameters to piers dimensions. In Castilla, a Spanish historic region, Friar Laurence of Saint Nicholas emerged as a conservative theoretician during 17th century due to the large amount of ruined bridges during this period. Regarding to arches, Friar Laurence recommended the round arch type and to add compacted soil to backfill the barrel. In addition, as mentioned by Huerta Fernández [1], [6], he recommended to fill up the inner part of piers using a specially compacted fashion, and continue this procedure up to 2/3 of the arch rise. This way to backfill became a custom in the country, as was verified during rehabilitation interventions in Spain. That is the reason why the 2/3 value has been used by researchers during structural analysis of masonry bridges, as in the case of González Parejo [7].

The emergence of analytic calculations provoked an extensive use of formulations during 18th century and later. Different bridge engineers proposed some expressions that were used to design multiple structures. Several of them are summarized in Table 2, including the mentioned by Oliveira et al. [8], Manjón [9] y Martín-Caro [10].

	Author	**Year**	**Semicircular or round arch**	**Segmental arch (reduced r/s)**	**Notes**
18th cent.	**Gautier**	1714	$b_t = s/18$		For s< 10 m and hard stone
			$b_t = 0.32 + 0.067 \times s$		For soft stone
	Perronet	1777	$b_t = 0.325 + 0.035 \times s$	$b_t = 0.325 + 0.0694 \times s$	
19th cent.	**Gauthey**	1809	$b_t = 0.33 + 0.021 \times s$		For 2 < s< 16 m
			$b_t = 0.0417 \times s$		For 16 < s< 32 m
			$b_t = 0.67 + 0.021 \times s$		For s > 32 m
	Sganzin	1809	$b_t = 0.325 + 0.3472 \times s$		

	Déjardin	1845	$b_t = 0.30 + 0.045 \times s$	$b_t = 0.30 + 0.025 \times s$	
	L´Eveille	1854	$b_t = 0.33 + 0.033 \times s$	$b_t = 0.33 + 0.033 \times \sqrt{s}$	
	Rankine	1862	$b_t = 0.19 \times \sqrt{s}$		
	Dupuit	1870	$b_t = 0.20 \times \sqrt{s}$	$b_t = 0.15 \times \sqrt{s}$	
	Croizette-Desnoyers	1885	$b_t = 0.15 + 0.142 \times \sqrt{s}$		
				$b_t = 0.15 + 0.183 \times \sqrt{s}$	For r/s=1/6
				$b_t = 0.15 + 0.206 \times \sqrt{s}$	For r/s=1/8
	Elzeario Boix		$b_t = 0.333 \times \sqrt{s}$		
	Luis Gaztelu. M de Echandía		$b_t = 1.35 + 0.75 \times \sqrt{s} - 1.40 \times \sqrt[3]{s}$		
20th cent.	**Séjourné**	1914	$b_t = 0.15 + 0.15 \times \sqrt{s}$	$b_t = 0.15 \times \left(1 + \sqrt{s}\right) \times \mu$	For $\mu = \frac{4}{3} \times \left(1 - \frac{r}{s} + \left(\frac{r}{s}\right)^2\right)$

Table 2. A set of expressions to calculate barrel or voussoirs thickness (b_t), in circular arches.

There were no specific differences between round arch and segmental arch until the 18th century, which can be understood as an indicator of the extensive use of the semicircular type as a trend. To check this hypothesis, the geometry of all masonry arches in the Carrion river basin has been analyzed (Figure 4). This means the study of 12 bridges and up to 101 arches. As shown in Table 3, most of them are included in the round arch typology. However, there was a trend change during 18th century, when segmental arches (including semi-shallow and shallow cases) emerged as a building possibility. In fact, most of arches built from 18th century were segmental arches.

	All arches		**Arches since 18th century**	
Arch type	**Number of arches**	**% of total arches**	**Number of arches**	**% of the typology**
Round arch	70	69%	23	33%

Semi-shallow	22	22%	21	**95%**
Shallow	6	6%	6	**100%**
Pointed arch	3	3%	0	0%

Table 3. Arch classification in the Carrion river basin rivers in function of type and period. Fourth column represent the number of arches built since 18th century. Fifth column shows the percentage of each typology built from 18th century. Note that most of segmental arches (semi-shallow and shallow) were built since this century (95 and 100% respectively, shadowed in the table).

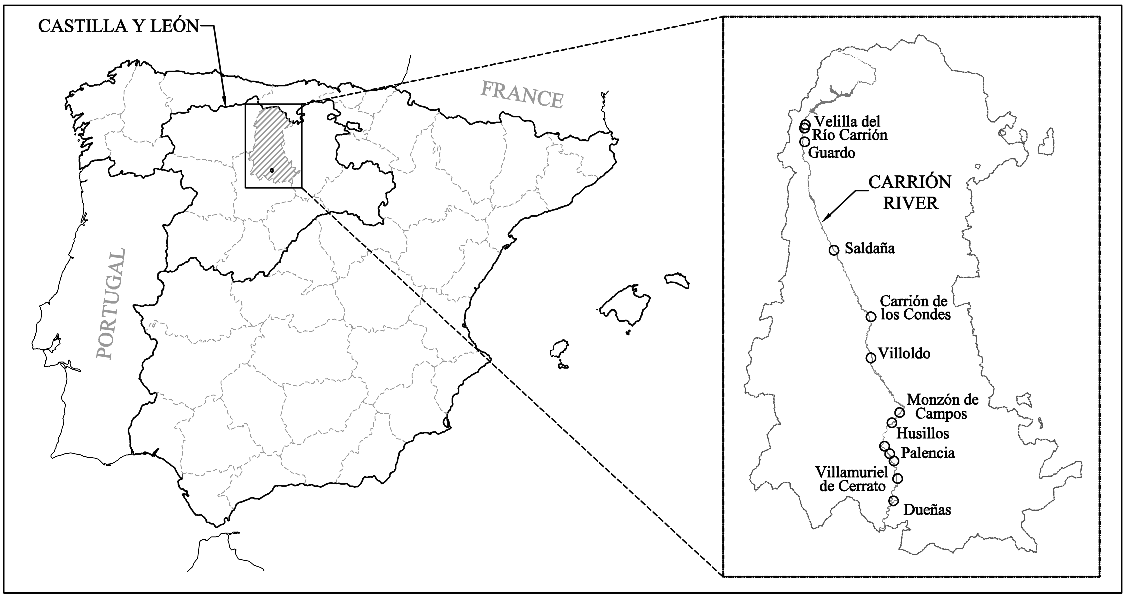

Figure 4. A general view of Carrion river basin location in Spain, including the position of the studied bridges.

When using expressions from Table 2, only Perronet distinguished segmental arches during 18th century. His proposal reduced voussoirs or barrel thickness when r/s grown, and in any case when s was larger (Figure 5). Nevertheless, Déjardin (19th century) pointed a paradoxical conclusion for this historical moment: segmental arches require lower barrel thickness than a round one for the same span length (Figure 5). Dupuit used similar recommendations later. When comparing Perronet and Déjardin expressions anyone can observe that a round arch (r/s=1/2) using the first formulation obtain similar values than Déjardin for a r/s=1/8.

At the endings of 19th century the shallow arch was extensively used, and formulations shown again a higher value for b_t for segmental arches, when compared with the semicircular case. Croizette-Desnoyers and Séjourné expressions can be used as example (Figure 6). In the case of Carrion river basin bridges, the most used was the Perronet formulation (Table 4). Nevertheless, there are two bridges built in the early 20th century using Séjourné expression and shallow arches. These are Velilla del río Carrión Bridge and San Isidro Bridge in Dueñas, both erected using similar features.

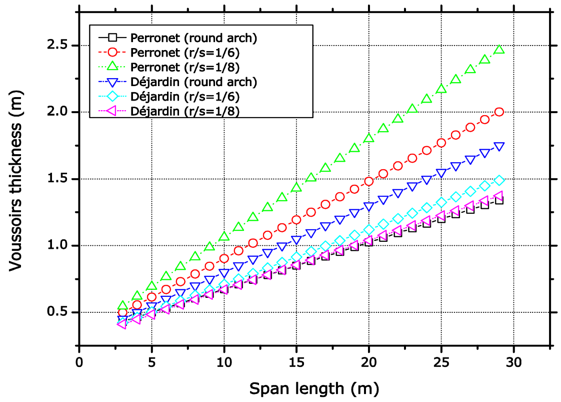

Figure 5. Comparative graph showing Perronet (1777) and Déjardin (1845) formulations for three r/s possibilities.

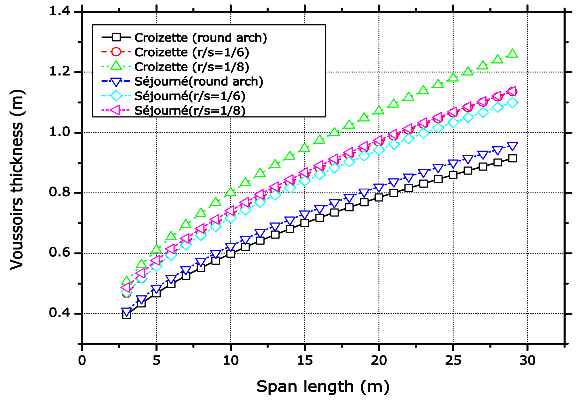

Figure 6. Comparative graph showing Croizette-Desnoyers (1885) and Séjourné (1914) formulations for three r/s possibilities.

3. SINGLE SPAN BRIDGE ANALYSIS

Several researchers have worked about different questions related with building material characteristics or structural response, to deep understand the behavior of masonry bridges. Among them we can cite, for example, Brencich y Morbiducci [11], and for the Spanish case Martín-Caro [10], González Parejo [7] or Ramos Casquero [12]. In the last one, the researching group found a clear conclusion: the load position, the presence of compacted backfill, and the r/s ratio are important parameters when measuring the ultimate load capacity of a masonry arch bridge.

Formulation	Number of arches	Number of bridges
Séjourné	6	2
Perronet	17	4
Gauthey	15	3
Gaztelu	10	2

Table 4. Summary and classification of all the bridges and arches in the Carrion river basin.

González Parejo [7] studied for Spanish examples the variation of ultimate load in function of load position, using a parametrical approach that revised geometries changing θ from 180 to 130 degrees. In other words, the study was performed for arches with r/s varying from 1/2 to 1/3.14. We have used this last study as starting point, increasing θ range and the number of analyzed variables. Therefore, multiple geometries were revised from r/s=1/20 to 1/2 (θ varying from 23 to 180 degrees). Our study was not focused on round arches, and the possibility of compacted soil was considered as well. Although irregularities in the barrel centerline definition can influence the value of stresses as shown by Morer y de Arteaga [13], for the purposes of our work we have use an idealized geometry to allow a comparative study.

For the calculations, we decided to use a software tool to enhanced accuracy. Particularly we have used Archie-M v 2.3.1, a code developed by Bill Harvey, Fraser Smith and Zoltàn Juhàsz in 2010 with the support of the Science and Engineering Research Council (EPSRC). This and previous versions of this software have been already used for masonry bridge analysis such as Kwooi-Hoch [14], and others that showed good correlation with experimental scale essays, as mentioned by Robinson [15]. The code carry out a 2D analysis by using the thrust line method. This computational program requires a few parameters to perform calculations, easing the inputs: geometry of barrel, piers and spandrel, load positions, specific weight. As a result of the calculations this software show a common collapse mechanism for this structures, a cinematic mechanism, as was revised by Morer et al[16] during a comparative analysis. Although it's true that different analysis procedures can be currently found using 3D arch definition and even take into account soil-structure interactions, as can be seen in Callaway et al [3], these complex approaches are more suitable to study specific parts of bridges, or the real collapse mechanism of a particular ruined example. Our objective was simpler: to carry out a generic parametric study that allows comparisons, in order to better understand the possible influence of certain variables related with structural response of an arch. In other words, we were not focused on stress concentration failures or subsequent deformations. The easiness of Archie-M, as mentioned by Kwooi-Hoch [14], was an important feature to decide its use.

All the calculations have evaluated a single 8 m span bridge with b_t=0.6 m, in order to quickly compare results for different r/s ratios. Other parameters of the model have been fixed before starting computational work. Firstly, the specific weight was 24 kN/m^3 for stone, 20 kN/m^3 for compacted soil backfill, and 18 kN/m^3 for standard soil backfill. As a default, and according to method principles, Archie-M does not allow relative movements between voussoirs, and tension strength is considered to be null. Compression strength was fixed to 30 MPa, as recommended by Holmström [17] and similar to those proposed by Martinez et al [18]. As proved by González Parejo [7], there is no significant influence in ultimate load when using compression strengths above 7.5-10 MPa. The nearly flat ultimate load values obtained for higher strengths was previously pointed by Holmström [17], and is coherent with values used by Rahman et al [19], with negligible effects for upper values.

Regarding to compacted soil backfill, we have followed the conclusions of González Parejo [7] based on studies carried out in the Civil Engineering School of Universidad Politécnica de Madrid. Accordingly, compacted soil behavior is like stone for macroscopic results. The same conditions have been previously used by Martín Caro [10], and were observed by Espejo y León [20] during the essays performed in Riera de Rubí Bridge and Plazaola Bridge in Urnietasu (Spain). In this last publication, authors observed and measured different variables of arches behavior during an extreme load until collapse. Their notes highlighted the compacted backfill importance in the final response, by means of providing a structural continuity of barrel towards piers.

Calculation procedure consisted in moving a point load all over the bridge deck, increasing value progressively, and after adding self-weight as dead load. The point load effect is treated internally by opening it using a vertical cone of 45 degrees of semi-angle. This value is in the range recommended by Rahman et al [19], and are coherent to those previously used in González Parejo [7] and Ramos Casquero [12]. Calculations were stopped when the thrust line emerged outside the barrel, considering this moment as a collapse and ultimate load is registered.

For all the already mentioned cases, the worst load position was determined as done by Audenaert [21]. In addition, the possibility of a 2/3 of rise compacted soil backfill presence was considered as well to perform a comparative analysis. As already said, compacted soil backfill influence is a key point in ultimate load evaluation, and used values are in the order of magnitude of other publications such as Callaway et al [3] , Bjurström y Lasell [4] y Cavichi y Gambarota [5].

3.1. **Compacted backfill and r/s influence during ultimate load evaluation**

Using a simple graphical approach (Figure 7), the reader can easily understand an initial mechanism that shows the important variation in arch behavior due to compacted backfill presence up to 2/3 of rise. Particularly for the round arch, the differences are remarkable. In this case arch becomes like a segmental arch with r/s near to 1/5. In other words, the barrel way of working substantially differs as can be observed when comparing ultimate load with and without compacted backfill (Figure 10).

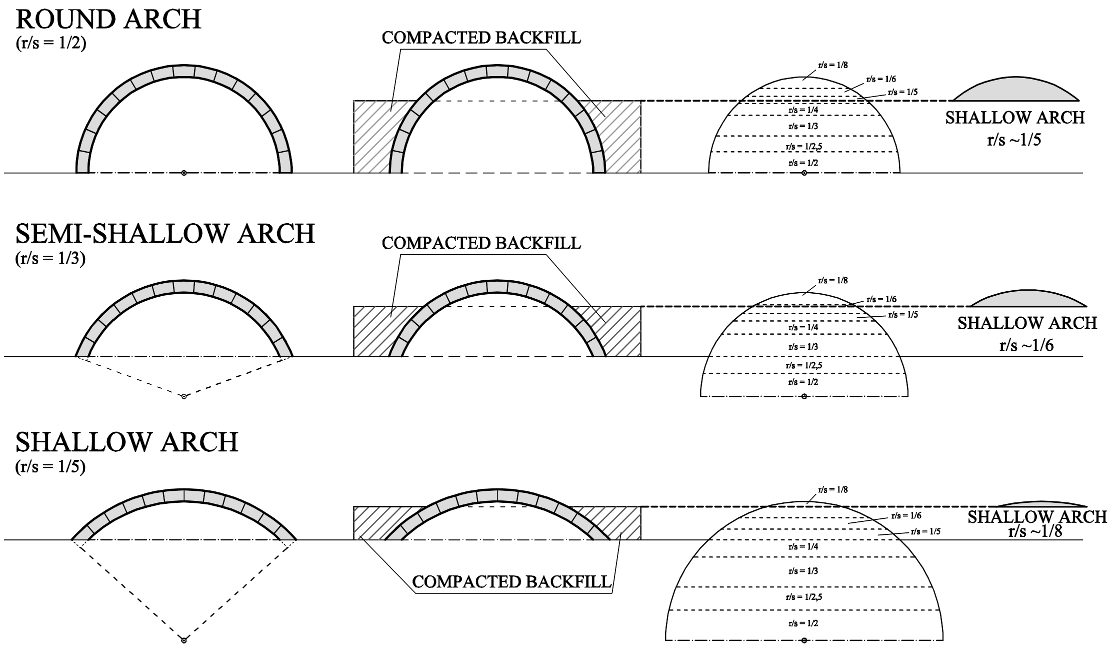

Figure 7. In function of the type of arch, compacted backfill may exert a huge influence in arch behavior.

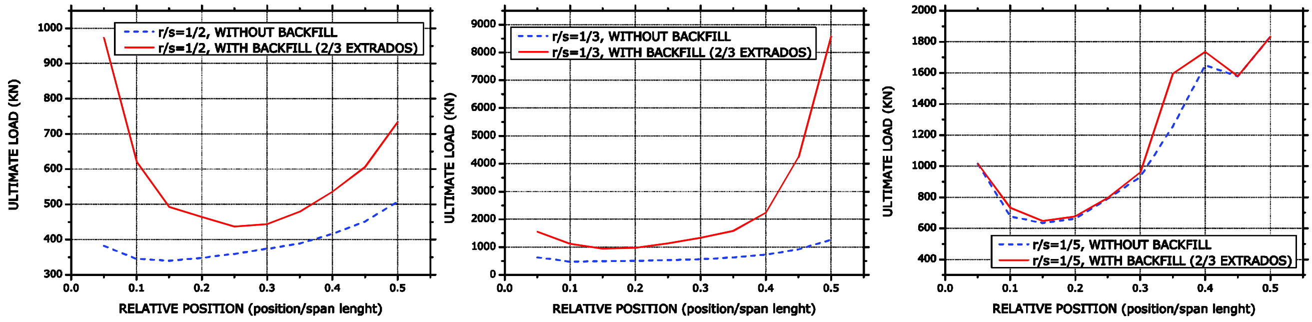

Figure 8. Variation of ultimate load with and without compacted backfill for three r/s assumptions: left r/s=1/2, center r/s=1/3, right r/s=1/5.

When passing r/s=1/5, backfill importance become negligible. That is the reason why differences between both curves are lower for higher r/s ratios and, in parallel, concavity suffers an attenuation. In the limit, which in fact represents a linear beam, curves are monotonically increasing as can be appreciated for r/s=1/20 in Figure 9.

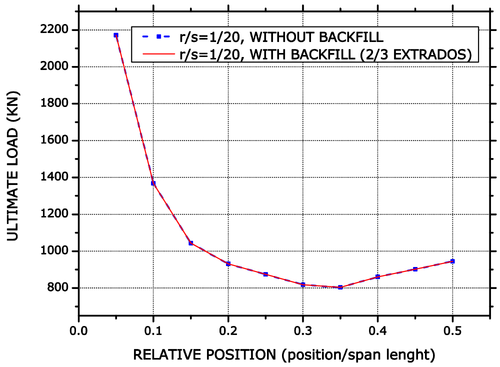

Figure 9. Ultimate load value (KN) in function of relative load position, with and without compacted backfill, for r/s=1/20.

Those typologies that are more influenced by backfill suffer a variation in their effective span length, understood as the distance between thrust line intersection with barrel boundaries (Figure 10). This provokes an increase of ultimate load values when backfill is considered and, hence, a peak in bridge strength. In a previous work [22], most unfavorable load position was computed for different r/s. Results showed that ultimate load was higher when backfill is considered for r/s up to 1/3 (Figure 11). That results were also obtained by González Parejo [7], but his study did not include lower r/s ratios. The extension of the study has shown that is precisely this value r/s=1/3 the one which leads to the highest possible ultimate load.

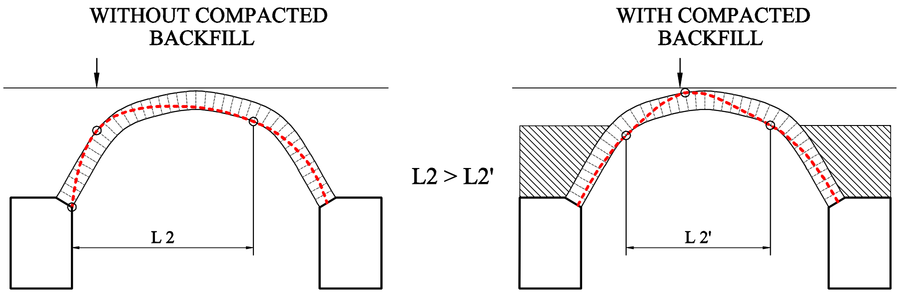

Figure 10. The picture shows the span length modification provoked by compacted backfill for those arches with r/s<1/3. The position of unfavorable load is also included.

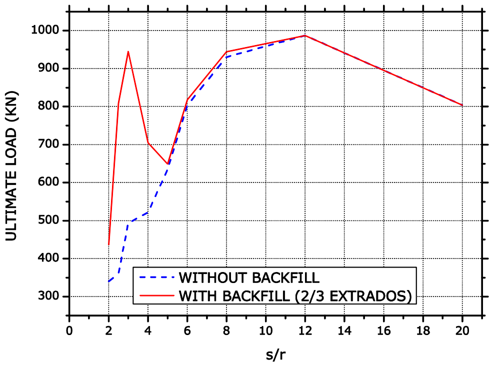

Figure 11. Summary graph that represents the ultimate load in function of r/s ratio, for a constant span length of 8 m. Both cases, with and without compacted backfill, are included for direct comparison.

Once passed this 1/3 limit, the flatter is the arch, the lower ultimate load is achieved as a result of calculations until reaching a second limit. In this case, a minimum is observed at r/s=1/5. But if we continue reducing r/s, the barrel acts nearly exactly as a without backfill, drastically reducing the previous influence. The explanation of this behavior requires considering a double effect: firstly, the thrust line does not touch anymore in inner part of the barrel, which means an increase of effective span length (Figure 10); on the other hand, there is a progressive increase of radius of curvature that, at the same effective length, becomes the determining factor. This allows give a reply to the paradoxical recommendation of Dupuit during 19th century (see section 2). One must note that, in this historical period, the common trend was precisely r/s=1/3, and hence a very good behavior should be perceived when using this typology.

Additionally, a minimum load envelope was graphically performed (Figure 12) to study the compacted backfill influence in the ultimate load in function of load position. Selected values were extracted for all the r/s possibilities already studied. The analysis showed a positive influence of the use of compacted backfill for all possible load positions, emphasizing this effect in positions near springers. The logic of this result relates to the backfill concentration zone, at spandrels. High ultimate loads are also obtained near keystone when compared with intermediate positions. The lower value is addressed for loads around 1/4 to 1/3 times s, which is coherent with González Parejo [7] and Robinson [15] when backfill is present.

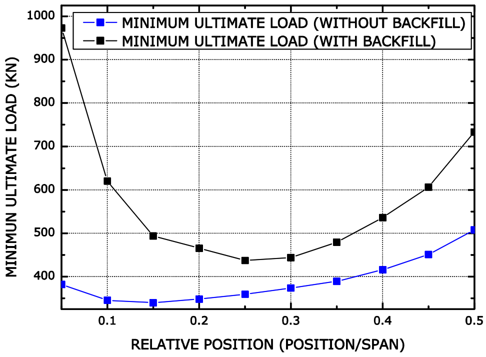

Figure 12. Minimum ultimate load envelope in function of load position, for all r/s ratios studied.

3.2. **Compacted backfill and r/s influence during maximum stress evaluation**

Although maximum stress is not the critic parameter for common loads in masonry bridges, it may become important for extreme loads. Accordingly, its variation has been studied as well. Using a similar procedure that explained for ultimate load, the analysis has included the presence of compacted backfill. The case was selected using the same geometry, and keeping the basic hypothesis. As our objective was a comparative analysis, the calculations have been performed using the standard point load used bay Archie-M, 112.8 KN, which in fact represent a commonly used load for bridge evaluation. The position of the load has been varied over the deck, including self-weight as well.

Results have shown that the positive effect of compacted backfill in arch strength become higher for r/s ratios near the round arch case (Figure 3). The backfill provokes changes in the effective length and in the real r/s way of work. Therefore, calculations allow similar conclusions than those obtained for ultimate load, stressing their coherence and justification. In fact, a higher influence of previously described factors is achieved in this case: the change in r/s (Figure 8) modifies the stress distribution y vary the arch way of work. That can be noticed when comparing the notable differences between the curves drawn with and without backfill (Figure 13). Secondly, backfill increases the effective span length (Figure 16) for the r/s=1/2 case. When decreasing r/s ratio, the opposite is observed, decreasing effective length. But another effect appears when comparing only those with backfill: the lower r/s, the higher effective length. These effects provoke a higher maximum stress for round arches in most load positions when backfill is present (Figure 15).

A summary of results is shown in Figure 15. The compacted soil backfill influence shows a change of trend for r/s above 1/2.5. From r/s=1/3, the maximum stresses at any load position are achieved for the arch without backfill. This trend continues for lower ratios.

In the case of no backfill present (Figure 15), the stress grows as the arch become flatter. In this r/s range the effective length increases until it reaches real span dimension (Figure 16). Moreover, tensional response shows fewer differences when compared with the presence of backfill, and even become negligible at los r/s ratios. The explanation is again related with a double effect. First, effective and real lengths became the same dimension (Figure 16) at a particular r/s ratio. Hence this factor is no longer present. Second, the behavior becomes homogeneous when the radius of curvature increases. In other words, the flattener becomes the arch, the lower influence has the compacted backfill because effective length is no longer reduced, which is emphasized with the lower change in r/s ratio.

Conclusions are coherent with those explained in section 3.1. In both studies, backfill influence is negligible for r/s lower than 1/5.

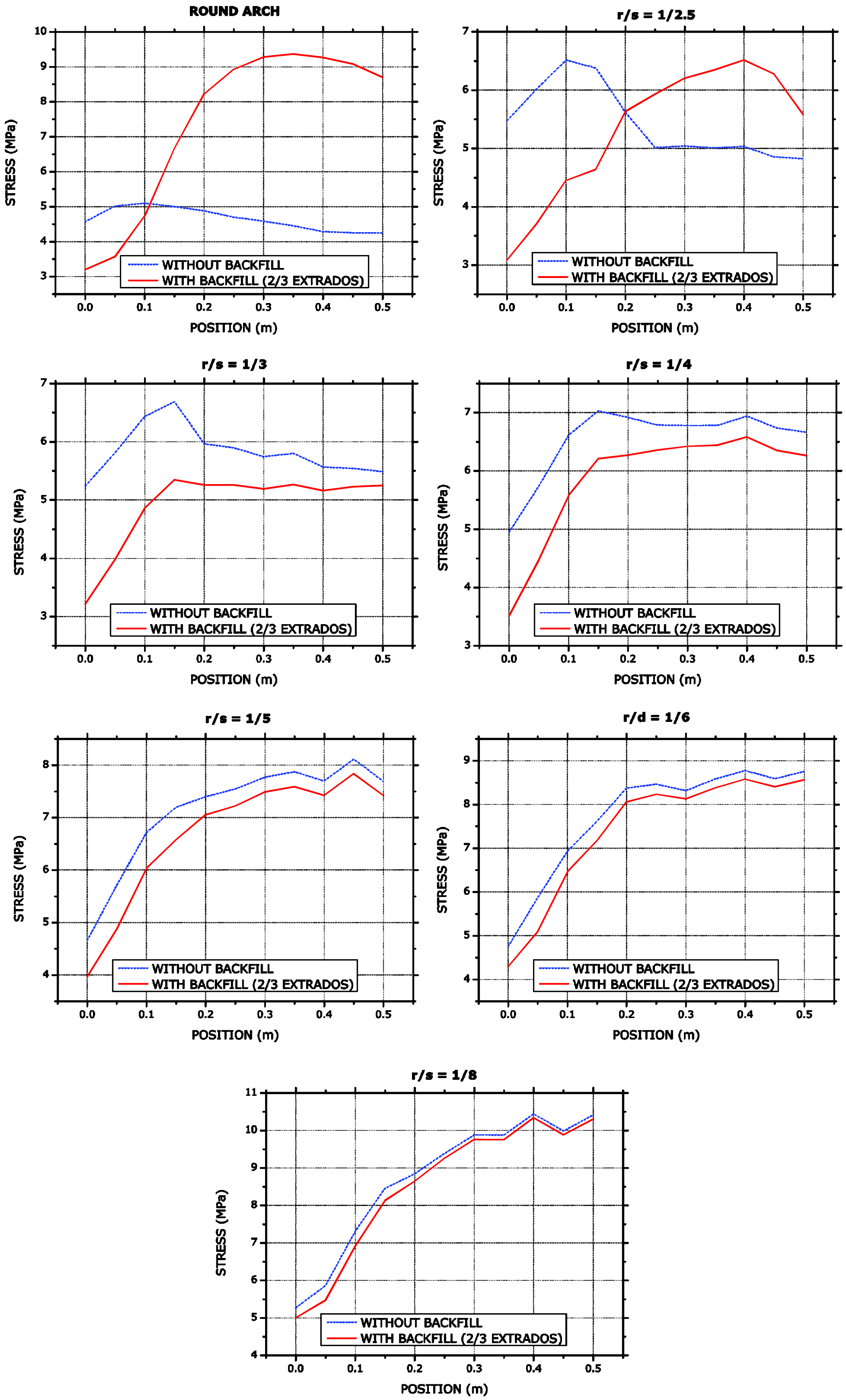

Figure 13. A set of graphs that show the maximum stress variation as a function of load position, including the possible backfill presence.

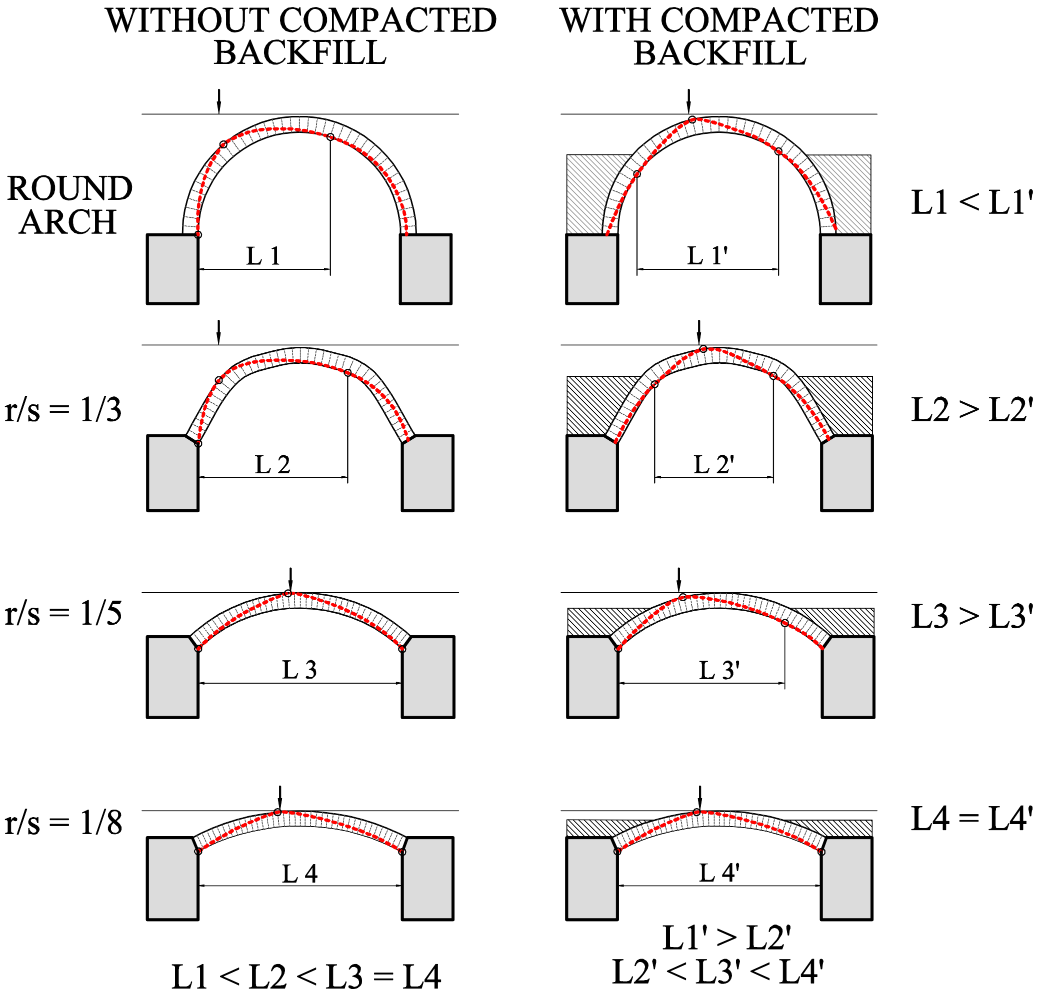

Figure 14. The pictures show the effective span length variation for the most unfavorable position at any case. Left column represents arches without backfill, and right column with the influence of compacted soil backfill.

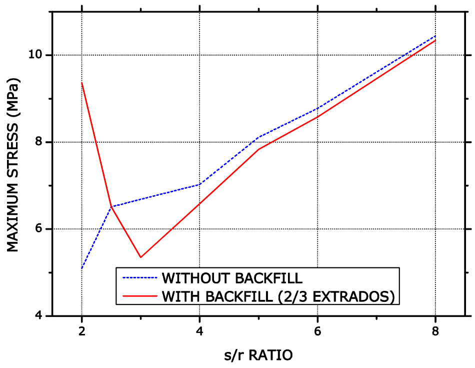

Figure 15. Summary of maximum stress for any r/s ratio, and at any load position. Calculations were performed for an 8 m span.

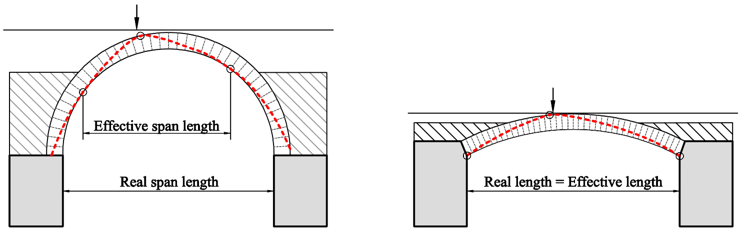

Figure 16. Compared results for thrust lines in the case of a round arch (left) and segmental arch with r/s=1/8 (right). The backfill is present in both cases. From a particular r/s ratio, effective length becomes equal to real span length.

4. VELILLA DEL RIO CARRION BRIDGE ANALYSIS

A real case bridge analysis was carried out to understand the change in building trends towards lower r/s ratios. Geometric data were acquired and presented in Table 5 and Figure 17.

Arch number	Arch shape	s	Barrel length	r	r/s	Pier number	Pier width
1	Shallow	12	5	2.22	1/5.4	1	2.5
2	Shallow	12	5	2.22	1/5.4	2	2.5
3	Shallow	12	5	2.22	1/5.4		

Table 5. Geometric data for Velilla del río Carrión Bridge (Spain). All magnitudes are expressed in meters.

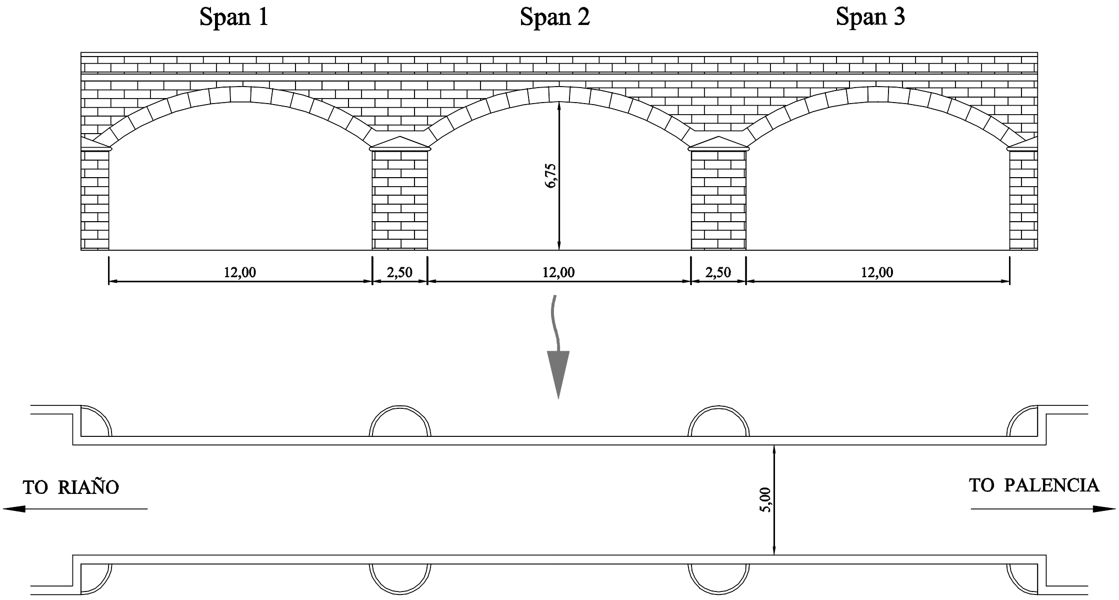

Figure 17. Ground plan and elevation of Velilla del río Carrion Bridge (Spain).

Bridge barrels were designed using Séjourné expression, but piers follow Perronet formulations. This is not an isolated example for the use of different formulations. Multiple examples using similar procedure were identified in several multi-span bridges in the north of Portugal and Spain, as shown by Oliveira, Lourenço y Lemos [8]. These authors explained this trend due to the previous background of builders, whose were used to erect robust piers following acquired experiences.

From a structural point of view, piers height influence is important for multi-span bridges as proved by Yang Yan [23]. Séjourné expression leads to slender piers for low heights, as occurs in the analyzed bridge. The designer should decide among the different formulations to project bridge dimensions. Why designer did not use Perronet for the arch, and a round type? Or why did not use Séjourné formulations for a round arch solution?

A multiple study was carried out using similar criteria already mentioned in section 3. The objective in this case was to understand if the selection was focused on structural variables, or otherwise there was an aesthetic motivation during the design.

Four different alternatives were calculated (Figure 18), keeping constant different magnitudes for comparison: deck height, pier width, and span length. Studied possibilities were as follows:

1- Round arches, using Perronet formulation for b_t, and leading to lower piers.
2- Shallow arches, using Perronet formulation for b_t.
3- Round arches, using Séjourné formulation for b_t, and leading to lower piers.
4- The real model of the bridge, qith shallow arches using Séjourné formulation for b_t.

For a shallow arch with r/s=1/5.4, Perronet expression produces a b_t near to 1 m, while Séjourné formulation gives 0.75 m. Besides, more similar are values computed by using Séjourné for shallow arch and Perronet for round arch, as shown in Table 6. To avoid the influence of pier height and barrel thickness, a new set of calculations have been performed using real value for them but varying r/s parameter. These results are shown in analysis number 1 and 4 to 7 in Table 6. The graphic representation allows to conclude that clearly the best values are obtained with r/s=1/3. However, the real r/s ratio produces a structural behavior near to optimal with an aesthetic design (Figure 19). Accordingly, the selection of a shallow arch allowed the evolution to more aesthetic shapes, but without renouncing to a good structural performance, near to the maximum.

The series of calculations produced the results summarized in Table 6, corresponding to analysis numbers 1 to 4. Selected typology, number 4, does not lead to the higher ultimate load, but allows a lighter configuration among the most strength.

Analysis	r/s	s	b_t	s/b_t	Expression	Type	Pier width	Pier height	Pier/s	Ultimate load (KN)
1	1/2	12	0.75	16	Perronet	Round	2.5	1.49	0.21	451
2	1/5.4	12	0.99	12	Perronet	Shallow	2.5	4.35	0.21	733
3	1/2	12	0.67	18	Séjourné	Round	2.5	0.82	0.21	395
4	**1/5.4**	**12**	**0.75**	**16**	**Séjourné**	**Shallow**	**2.5**	**4.52**	**0.21**	**564**
5	1/3	12	0.75	16		Semi-shallow	2.5	4.52	0.21	606
6	1/8	12	0.75	16		Shallow	2.5	4.52	0.21	437
7	1/12	12	0.75	16		Shallow	2.5	4.52	0.21	303

Table 6. Comparative table including geometric features and ultimate load for the studied alternatives. The real built bridge configuration is shaded (alternative 4).

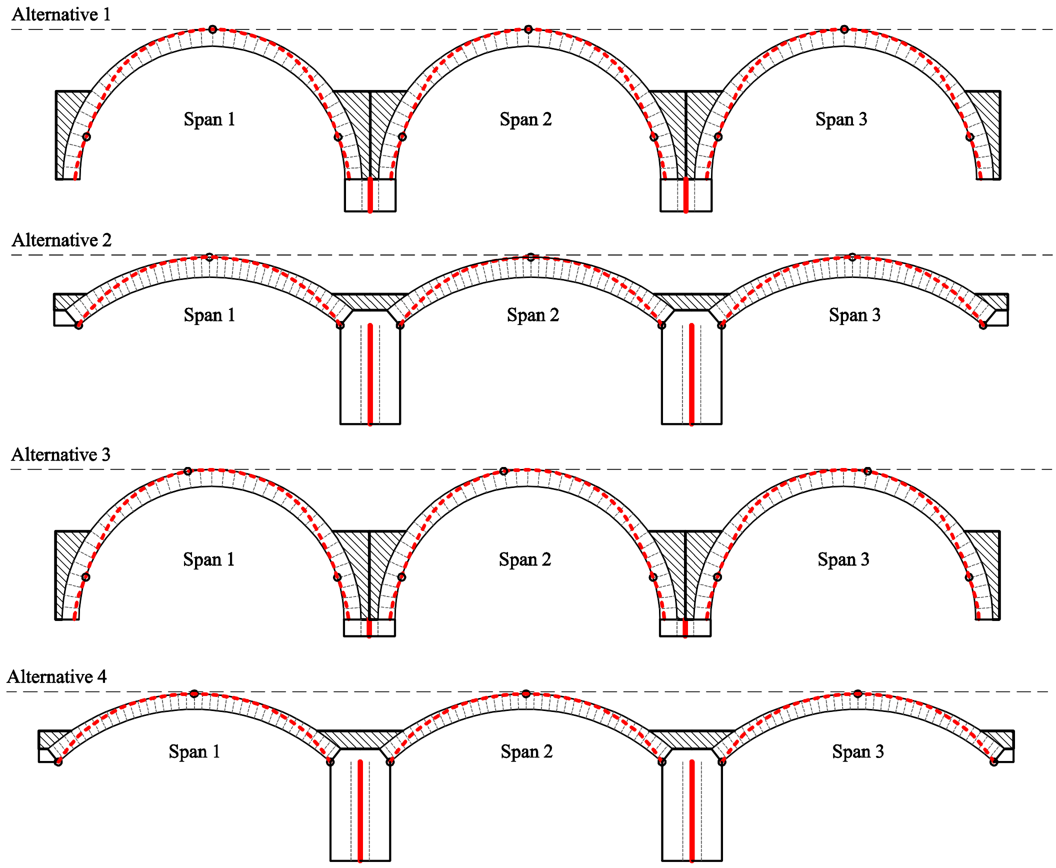

Figure 18. Elevation sketches of the four studied alternatives for Velilla del río Carrión Bridge.

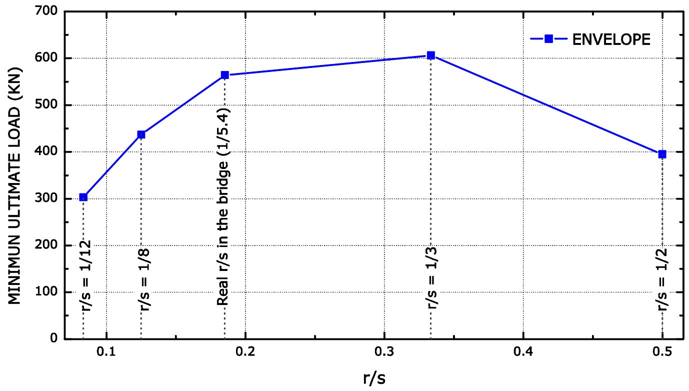

Figure 19. Minimun ultimate load capacity envelope for all Velilla del río Carrion Bridge alternatives, using different r/s.

5. CONCLUSIONS

Different studies have shown an extensive use of round arches in bridges of several Spanish regions. Although round arch represents the biggest percentage of designed bridges, there is a trend of variation towards segmental arches at the endings of 18th and early 19th centuries. These observations can be seen in several works devoted to Spanish basins, such as Arlanza river by Manjón [9], the province of Cáceres by por González Parejo [7], or our work in the Carrion river [24]. The series of analysis in this paper have shown that both ultimate load and maximum stress in masonry arches are strongly influenced by the presence of compacted soil backfill. These results were obtained varying the load position, the r/s parameter, and checking differences with and without the already mentioned backfill. Differences are clearly important for the round arch case, becoming lower as decreasing r/s.

A particular analysis has shown a maximum load capacity for r/s=1/3, when the backfill is included in calculations. This affirmation improves previous conclusions by González Parejo [7], as we have observed that for r/s>1/3 the influence of backfill finally becomes negligible. To explain this behavior, we have to combine a double effect. First, the higher r/s, the higher is the change in the structural way of work when adding compacted backfill. On the other hand, the reduction of effective span length produced by the backfill reaches a minimum at r/s=1/3, but increasing for higher ratios.

In addition, differences in structural response with and without backfill (Figures 11 and 15) become lower as the r/s decreases. When a particular r/s is reached, effective and real span length become equal (Figure 16); besides, there is a response homogenization in both cases as the radius of curvature is increased. Namely, as the barrel takes a flat shape, there is a lower influence of backfill because the thrust line does not reduce effective span length and the change in the way of work is not appreciable. In any case, compacted backfill exerts a positive effect independently of load position. Therefore, the set of performed analysis has shown the statistical trend of the arch shape evolution from the round arch to the segmental one. Changes were produced from the endings of 18th century, and mainly during first part of 19th century. Produced results have explained that these changes in shape could be led by a best structural performance of segmental arches with compacted backfill, as formulations proposed by Déjardin and Dupuit tried to point out, and including a remarkable aesthetic improvement. To check this possibility, the real case of Velilla del río Carrión Bridge was tested using four different alternatives. Calculations in this last case shown that its r/s ratio (corresponding a shallow arch with r/s=1/5.4, very similar to other in the same basin) has a structural behavior near to optimal (which occurs at r/s=1/3) but with an evolutionary flat shape.

On the other hand, the size of piers carries lot of weight in the structural response of a multi-span masonry bridge, as proved by Yang Yan [23]. The analyzed bridge does not follow the same formulation for barrel and piers, in the same way that explained by Oliveira, Lourenço y Lemos [8] in a study of multiple bridges in the north of Portugal and Spain. The trend was to build stronger piers, maybe characteristic of previous periods. If the designer has used the Séjourné expression for piers, as did for barrel, results would have been too slender due to their low height. Hence, the standard was trying to build strong piers to last even at bad stream conditions, but designing aesthetic arches with lower r/s ratios.

In summary, among the studied variants to Velilla del río Carrión Bridge, the selected by builder merged in the solution the most possible aesthetic shape that allowed good structural performance. In fact, ultimate load of real bridge is near the optimal as showed in Figure 19.

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