# Effects of fiber material in concrete manufactured with Electric Arc Furnace Slag: experimental and numerical study

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Preprint submitted to Construction and Building Materials

23 experimental campaign and, secondly, a numerical simulation to model the 24 effect of fibers both in the pre-cracking and post-cracking stages. Importantly, 25 for the numerical study, an in-house Finite Element (FE) code is developed 26 using interface elements to capture crack propagation. The FE code uses, as 27 input, data obtained in the experimental campaign and is validated against 28 previously unseen experimental results. The overall framework gives 29 important insights on how fibers improve the post-cracking behavior of EAFS 30 concrete and the relevance of fiber material in the overall performance. The 31 validated numerical tool can be used in the future to design EAFS fiber-32 reinforced concrete structures and therefore increase the applicability of 33 such composite material.

Keywords: Electric Arc Furnace Slag, Steel/Synthetic fibers, Dog-bone test,
Interface solid finite elements, Tensile damage models.

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#### 1. Introduction

38 Concrete is one of the most widely manufactured materials across the 39 world and the expansion of the concrete industry continues apace. It is 40 reported that annual concrete production worldwide increased from more 41 than 10 Gt in 2006 [1] to 32 Gt in 2017 [2]. Set alongside the climate 42 emergency context [3] there is a pressing need to improve the sustainability 43 of concrete production. If that objective is to be fulfilled, then energy-saving 44 techniques, lengthier service lives, and reusability/recycling of materials are 45 among the factors that must therefore be given serious consideration.

In this context, many researchers [4–12] have studied the use of Electric
Arc Furnace Slags (EAFS), an industrial waste generated during the
steelmaking process, in the replacement of natural aggregates. Reusing EAFS
reduces the amounts of waste buried in landfills, while improving certain
features of the concrete [12–16].

51 In a positive sense, EAFS concrete shows a better mechanical performance 52 than other traditional concretes [10,12,16,17]. EAFS high porosity and 53 roughness provides a strong interlocking effect between aggregate and 54 matrix [5,6,12,13,15]. Additionally, the greater durability of EAFS concrete 55 has therefore extended its structural service life [10,13,14]. In a negative 56 sense, the specific density of EAFS is about 15 % higher than natural 57 aggregates, due to metallic inclusions within the slags [18]. Although 58 advantageous for some applications where weight is a key factor (sea-walls, 59 foundations, ballast, etc.), its use is a downside to get self-compacting 60 condition [11,19,20]. Engineers have to take account of the weaknesses of 61 EAFS concrete, just as they have to do for concrete made with natural 62 aggregates, such as brittleness, poor tensile strength, poor resistance to 63 impact strength, fatigue, and low ductility [21]. Previous studies [17,22–24] 64 have proven that additions of randomly distributed small fibers help to 65 address some of these weaknesses. The main role of fibers is essentially to delay crack propagation across the matrix by bridging the crack tips 66 67 [23,25,26]. However, fiber additions provoke certain problems for mixing and 68 workability. Fibers can show a tendency to clump together, forming balls, or 69 to distribute themselves in non-uniform ways, thereby altering the properties 70 of the composite [14,27,28]. These effects, together with the irregular sizes 71 and shapes of EAFS, makes fiber-reinforced EAFS concrete a heterogeneous 72 material with large inter-sample variability. Its overall reliability is therefore 73 affected, presenting significant obstacles to its widespread adoption for 74 structural purposes [29].

There is therefore a pressing need for a better understanding of the mechanical behavior of EAFS fiber-reinforced concrete. This area has been explored by several authors [14,17,18,29–31], including our recent contributions in [32,33], where it was found that bending strength and postcracking behavior were improved through the use of EAFS in replacement of

80 natural aggregates and the addition of fibers. Even though very useful insight 81 into the mechanical behavior of the material has been gained from 82 experimental studies, the results are always limited to a specific set of 83 measurements and involve significant time and economic constraints. 84 Limitations that become all the more apparent when large structural 85 components are involved. Computational modeling, and specifically FEA can 86 contribute very accurate quantitative information, such as displacement, 87 stress, and fracture. Additionally, once properly developed and validated, FEA 88 can act as a predictive tool which can be used for the design of structural 89 elements

90 As far as the authors are aware, no FEA model has been provided for EAFS 91 fiber-reinforced concrete elements and synthetic fiber-reinforced concrete. 92 Building upon our recent experimental contributions [18,32,33], this paper 93 aims to provide an experimental and computational framework to develop an 94 accurate FEA model of EAFS fiber-reinforced concrete including fracture 95 effects. The framework that the authors of this paper have developed not only 96 comprises an accurate finite element model of EAFS fiber-reinforced concrete 97 deformation and fracture, but also a detailed experimental campaign, on 98 small samples, to gather all the necessary model input data. The developed 99 in-house FEM is based on elements with high aspect ratio (interface 100 elements) where tensile damage models are implemented, to describe the 101 behavior of the composite. Interface elements and tensile damage models are 102 capable of capturing concrete fracture and fiber bridging effects, due to steel 103 fiber additions, as also shown in previous research [25,34,35]. In this 104 investigation, the performance of interface elements for modeling synthetic 105 fibers was also analyzed and the numerical framework was validated against 106 the experimental data.

107 The paper will therefore be organized as follows. Section 2 introduces the108 numerical and experimental framework giving a brief overview of the tested

material, experiments and FEA. In Section 3, the experimental results will be
presented and the mechanical performance of steel/synthetic fibers will be
assessed. In Section 4, the numerical results will be reviewed and, finally,
some concluding remarks will be given in Section 5.

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#### 114 **2. Experimental and numerical framework**

115 *2.1. Materials* 

In this research three different mixes are analyzed, in order to study the
effects of steel and synthetic fibers on EAFS concrete. The mixes were labeled
following the presented nomenclature in [16,32] as: IISC (plain concrete),
IISCM (steel fibers) and IISC-Y (synthetic fibers).

The major difference between them concerns the addition of fibers, as is shown in Table 1. Type II cement was used in every mix and high-range water reducing admixture (superplasticizer) was also employed to improve workability and mechanical properties. Not all admixtures are compatible with every cement or fine aggregate and they can cause flowability problems, anomalous rheological behavior or just, not achieving the desired properties [36]. Its compatibility was previously studied in [16].

127 EAFS were added in two different grading, 4-12 mm and < 4 mm with a 128 fineness modulus of 5.7 and 3.9, respectively. In the absence of fine aggregate 129 (<1.2 mm) due to high energy required to crush EAFS, limestone (95 % of 130 calcite) with a fineness modulus of 1.5 were added to increase the 131 cohesiveness of the paste [16,37]. This grading enabled to achieve the 132 required characteristic in terms of self-compactness and strength. The EAFS 133 were previously subjected to an aging process that consists of irrigating and 134 moving the slag in order to provoke the hydration and carbonation reactions 135 of the possible expansive components presented in it. In this way, the slags 136 maintain its volumetric stability for its application in concrete [6]. The stability of EAFS aggregate was verified by ASTM D-4792 test [38], prolonging
the prescribed test duration over 90 days; the results ensured slag expansion
smaller than 0.5 %. The design strength of concrete was 40 MPa, as good
quality mixes for structural elements, employing a moderate content of
binder per cubic meter of concrete.

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#### Table 1: Concrete mix proportions (Kg/m<sup>3</sup>)

Constituents	IISC	IISC-M	IISC-Y
CEM II/B-S 42.5R	330	330	330
Admixture	5.3	5.3	5.3
<b>EAFS:</b> <i>φ</i> = 4 – 12 <i>mm</i>	750	750	750
<b>EAFS:</b> $\phi < 4 mm$	550	550	550
<b>Limestone:</b> $\phi < 1.2 mm$	950	950	950
Water	170	180	185
Steel fibers	-	40	-
Synthetic fibers	-	-	4.5

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In this study fibers were also added (0.5 % of total volume) in two mixes (IISC-M and IISC-Y) to improve mainly the post-cracking expertise. Two types of fibers were used to improve the performance i) Hooked-end steel fibers (IISC-M) ii) Dimpled-surface synthetic (polyolefin) fibers (IISC-Y). The fact that the designed mixes are self-compacting (absence of vibration) enables not to influence in fiber distribution and direction, which ensures a fairly uniform fiber distribution.

152 Extensive analyses of these sorts of mixes may be found in previous works153 [16,18,32,33].

#### 154 *2.2. Experiments*

The mechanical properties of the mixes were defined through compression, three-point bending, and tensile tests. The test results are used to define the required inputs by the numerical model and to validate the proposed framework. The properties defined by the proposed set of experiments were:

- Compression test: Compression strength (f<sub>c</sub>), compression elastic
   modulus (E<sub>c</sub>) and the Poisson's ratio (v).
- Three-point bending test: The CMOD/Load curve of sample IISC
   was used to compute the fracture energy (G<sub>F</sub>) and the results of
   IISC-M and IISC-Y were used to validate the framework of tests that
   has been proposed.
- Tensile test: Direct tensile strength (ftd) and indirect tensile
   strength (fti).

Compression and bending tests are well-known and they were performed
according to the specifications described in the European standards [39–41].
However, tensile strength is not a frequently defined property for concrete. It
can either be determined by an indirect tensile test (Brazilian test) or a direct
tension test (Dog-bone test) [42–45].

The Brazilian test (or splitting tensile test) is a well-known method for
determining tensile strength through compressive loading. It was performed
in accordance with UNE-EN 12390-6 [46].

Direct tensile tests are hardly used for concrete and there are insufficient standards or instructions on the performance of these tests [45]. In this research, the direct tension test was performed using small size Dog-bone shaped specimens subjected to direct monotonic tensile loading. The dogbone specimen was designed based on the dimensions proposed in [45]. The overall length of the specimen is 164.1 mm and the cross-section at the head of the specimen was 55.5 x 30 mm, as shown in Figure 1. These dimensions,
together with the clamping jaws, mean that the specimen can be easily aligned
with no undesired rotations or constraints. These features are useful for
capturing uniaxial tensile behavior in the narrow section. Lastly, the Dogbone molds with the above measurements were built using a 3D printer.



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Figure 1 Dog-bone test set up (mm)

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#### 191 2.3. Finite Element Analysis: Interface elements

192 Discontinuities are generated in solids when they are loaded beyond their 193 elastic limits. From a mechanical point of view, they can be modeled as weak 194 145 or strong discontinuities. A weak discontinuity is characterized by a 195 continuous displacement field and discontinuous strain. In contrast, the 196 strong discontinuity kinematics is characterized by unbounded strain field 197 along the discontinuity surface. The modeling of strong discontinuities is 198 common in elements with irregular displacement fields and highly localized 199 failures [47] such as in brittle materials (EAFS concrete).

A technique based on the insertion of interface elements with a high aspectratio between regular elements of the mesh is used to describe the kinematics

202 associated with the discontinuities [34,45]. In this method, a solid is therefore 203 idealized as a two phase -interface elements and bulk elements- composite, 204 as illustrated in Figure 2. Bulk elements are considered elastic elements and 205 interface element behavior is governed by a softening law, which models 206 concrete fracture or bridging phenomena. Crack openings are simulated by 207 the degradation of interface elements. The governing equation in linear 208 elasticity (weak form) provides an approximate solution to the structural 209 mechanics of the composite:

$$-\mathbf{W} \cdot \int_{\Gamma} \boldsymbol{\sigma} \mathbf{n} \, \mathrm{d}\Gamma + \int_{\Omega} \boldsymbol{\sigma} : \nabla \mathbf{W} \, \mathrm{d}\Omega = \mathbf{W} \cdot \int_{\Omega} \boldsymbol{f}_{\boldsymbol{b}} \, d\Omega \tag{1}$$

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Figure 2: 2D Interface element

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214 where, **W** is the weight function, *o* is the stress field, **n** is the normal vector,

215  $\Gamma$  is the boundary,  $\Omega$  is the surface of the solid and  $f_b$  are the body forces.

The triangular element is geometrically defined by the nodal coordinates, which are used to calculate the normal vector, **n**, to the base, b, and the height of the element, h, as shown in Figure 2. Once the geometry is defined, the strain tensor ( $\epsilon$ ) can be written as the summation of the components of the strain tensor that depend on h ( $\hat{\epsilon}$ ) and on b of the triangle ( $\tilde{\epsilon}$ ):

$$\epsilon = \tilde{\epsilon} + \hat{\epsilon} = \frac{1}{b} \begin{bmatrix} 0 & \frac{1}{2} \left( u_n^{(3)} - u_n^{(2)} \right) \\ \frac{1}{2} \left( u_n^{(3)} - u_n^{(2)} \right) & u_s^{(3)} - u_s^{(2)} \end{bmatrix} + \frac{1}{h} \begin{bmatrix} \llbracket u \rrbracket_n & \frac{1}{2} \llbracket u \rrbracket_s \\ \frac{1}{2} \llbracket u \rrbracket_s & 0 \end{bmatrix}$$
(2)

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223 where, $u_n^{(i)}$  and  $u_s^{(i)}$  are the normal and tangential displacements and  $[\![u]\!]_n$ 224 and  $[\![u]\!]_s$  are the relative displacements of node 1 and its projection onto the 225 base, 1'.

Interface elements can be of either zero thickness or very little height (h  $\rightarrow$ 0). This condition indicates that  $\tilde{\epsilon}$  tends to be infinite (unbounded), as might be deduced in Equation 2. Therefore, the strains are almost dependent on  $\tilde{\epsilon}$ . The kinematics of the solution is conditioned mainly by the relative displacements between node 1 and its projection 1', h [34,35].

Then, the corresponding stresses can be calculated by the constitutive models, even though the strain tensor is unbounded. The tension damage model used to describe the behavior of interface elements is as follows:

$$\boldsymbol{\sigma} = (1 - d) \, \mathbb{C} : \boldsymbol{\epsilon} = (1 - d) \, \mathbb{C} : (\boldsymbol{\tilde{\epsilon}} + \boldsymbol{\hat{\epsilon}}) \approx (1 - d) \, \mathbb{C} : \, \boldsymbol{\hat{\epsilon}} = \frac{1 - d}{h} \mathbb{C}$$

$$: (\mathbf{n} \otimes \llbracket \mathbf{u} \rrbracket)^{s}$$
(3)

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where, d is the damage parameter,  $\mathbb{C}$  is the elastic tensor, and ()<sup>s</sup> refers to the symmetric part. The damage criterion,  $\phi$ , is defined in terms of equivalent stress,  $\tilde{\sigma}$ , and a stress-like internal variable q(r):

$$\phi = \tilde{\sigma} - q(r) \le 0 \tag{4}$$

The equivalent stress is computed through the stresses at the base of the triangle while q(r) synthesizes the softening behavior of the composite. The adopted bridging law is used to represent traction-separation laws for different steel-fiber-reinforced concrete in [25]:

$$q(r) = (f_{t,com} - t_1) e^{\frac{-r}{\omega_{ref}}} + t_1 \frac{\omega_u - r}{\omega_u} + t_2 r e^{c_1 - c_{2r}}$$
(5)

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where, r is a strain-like internal variable,  $f_{t,com}$  is the tensile strength of the composite,  $t_1$ ,  $t_2$ ,  $c_1$ , and  $c_2$  are the fitting coefficients,  $\omega_{ref}$  is the reference crack opening displacement, and  $\omega_u$  is the ultimate crack opening.  $f_{t,com}$  is derived from the tensile tests. In contrast,  $\omega_{ref}$  is determined through fracture energy,  $(G_F)$ , calculated from the three-point bending test applied to plain concrete (IISC).

$$\omega_{\rm ref} = \frac{f_{\rm t}}{G_{\rm F}} \tag{6}$$

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The first term of Equation 5 is associated with the fracture of plain concrete. The second term introduces the frictional aspect during the pullout procedure. The last term is correlated with the anchorage effect that some fibers might have due to their shape, such as the hooked-end fibers.

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#### 3. Experimental test results and discussion

The properties defined through the proposed tests are listed in Table 2 (mean values and the standard deviation in parentheses). The addition of steel fibers has hardly any effect on the compressive strength, as other authors have also concluded [17,32,48]. The tensile elastic modulus tended to be slightly higher than the compressive elastic modulus for the mix with

264	fibers, as also found in previous studies [49]. The Dog-bone test provides
265	lower tensile stress than the Brazilian test as previous studies suggested [50].
266	The values of uniaxial tensile strength were 17 %, 22 % and 16 % lower than
267	the indirect tensile strength for IISC, IISC-M and IISC-Y mixtures, respectively.
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Table 2: Compressive, tensile and three-point bending test results.

Property	IISC	IISC-M	IISC-Y
f <sub>c</sub> (MPa)	59.66 (5.7)	53.09 (1.5)	46.08 (1.0)
Ec(GPa)	40.1 (0.7)	34.7 (1.5)	31.6 (0.9)
ν	0.23	0.22	0.22
f <sub>td</sub> (MPa)	4.25 (0.2)	3.77 (0.4)	3.66 (0.4)
f <sub>ti</sub> (MPa)	5.11 (0.5)	4.84 (0.6)	4.35 (0.4)
$E_t$ (GPa)	38.5 (1.0)	37.9 (2.8)	35.5 (0.3)
G <sub>F</sub> (N/mm)	0.137	2.235	0.598

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272 Figure 3 shows the stress-strain curve of the Dog-bone test of the three 273 mixes. The samples were preloaded in 6 cycles of loading/unloading which 274 enables to assess the hysteric behavior of the mixes. In Figure 3(a) is more 275 evident that the slope of the loading/unloading curves is reduced which 276 means the material is damaged and the stiffness is reduced. Mixes with fibers 277 (IISC-M and IISC-Y) exhibit an improved hysteretic response in terms of 278 stiffness which increase the energy absorbing capacity. Despite possible size-279 effect uncertainties [51,52], these results are aligned with the expected 280 tensile behavior [49,50]. Although more tests are still required to provide





fibers.

Figure 3: Strain-stress curve of Dog-bone test.



284 The tensile strength of concrete is usually defined through empirical 285 relations based on compressive strength. In Figure 4, empirical curves for 286 plain concrete suggested by International Federation for Structural Concrete 287 (CEB-FIB) [53] and American Concrete Institute (ACI-318) [54] are drawn 288 together with the empirical equation proposed for fiber-reinforced concrete 289 by *Xu et al.* [55]. All the mixes are consistent with the proposed equations with ratios ranging between 7-12 %. Tensile strength is often approximated as a 290 291 tenth of compressive strength [56], which is also applicable in fiber-292 reinforced EAFS concrete.

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Figure 4: Relationship between tensile strength and compressive strength.

Figure 5 illustrates the CMOD/load curves defined in the three-point beam tests, in accordance with [41]. The cross-sectional dimensions of the notched beams were 150 mm by 150 mm, each with a length of 600 mm and a span of 550 mm. The notch was marked to a depth of 25 mm at mid span on the bottom side of each beam.



Figure 5: Average CMOD/Load curves of the notched beam test.

Fracture energy ( $G_F$ ) is a parameter used to model the post-cracking behavior of concrete. It is computed with the model proposed by *Hilleborg* [57], using the load/CMOD curve. Fracture energy dissipated up to a CMOD of 3.5 mm is reported in Table 2. From a design point of view, it is assumed that no further energy could have been absorbed after that point [26].

310 Figure 5 shows the load/CMOD curves of IISC, IISC-M, and IISC-Y. Focusing 311 on the pre-fracture behavior (damage stage), IISC-M shows slightly greater 312 values of load than IISC in line with the behavior reported in the tensile tests. 313 IISC-Y shows slightly greater values of load than IISC, in line with the behavior 314 reported in the tensile tests. IISC-Y shows lower values than plain concrete 315 that is consistent with the worst mechanical performance shown in Table 2. 316 This last difference might be due to factors relating to the manufacturing 317 process that increased the air content of the mixes [22] and the lower stiffness 318 of the synthetic fibers [17,22,58].

Beyond the first peak, the fibers play an increasingly prominent role and their effects become more evident, mainly depending on the dosage and properties of the fibers [23,59]. IISC showed a brittle behavior, while the fiber mixes maintained some residual strength after the crack, so their behavior was more ductile than the former. Three notched beams were tested for IISC-M and IISC-Y. IISC-Y samples showed very similar behavior while IISC-M presented higher variability between the samples (see Figure 8). This
variation is related with the uncertainties introduced when fibers are added
to the mix [59]. The improvement in the post-cracking behavior of IISC-M
became particularly relevant, multiplying by 3-6 the residual strength of mix
IISC-Y.

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### 4. Finite Element Analysis

332 The notched beams (IISC-M and IISC-Y) are numerically analyzed in this 333 section, using the Finite Element algorithm described in section 2.3. To do so, 334 the FE model of the notched beam is firstly defined in section 4.1. Secondly, 335 the numerical model is validated against experimental data in section 4.2. 336 Finally, in section 4.3, the validated model is used to extract physical 337 quantities that allow improving the insight into the fracture mechanics of the 338 fiber-reinforced beams. Such physical quantities would otherwise be 339 impossible or too difficult to estimate from experimental measurements.

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#### 341

#### *4.1. Definition of the Finite Element Model*

342 Figure 6 illustrates the finite element model and the boundary conditions 343 of the notched beams presented in Section 3. The numerical analysis is carried 344 out in 2D and plane stress conditions are considered. An unstructured mesh 345 is used to reduce the dependency of the cracking path with the mesh. It is 346 meshed with linear triangular elements of 10 mm. The mesh is refined (1 mm) 347 near the notch where the crack is expected to develop, to capture the cracking 348 more accurately. Table 3 lists the material properties of the bulk elements 349 defined through the tests while interface elements (0.001 mm) properties are 350 reported in Table 4. Interface elements inputs are defined by fibers properties 351 except for Poisson's ratio, which is assumed as null to introduce the discrete 352 relation between nodes based on Young's modulus [35].



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Figure 6: Finite element model of notched test beams (mm).

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Table 3: Material parameters used to model Bulk elements.

INPUT	IISC-M	IISC-Y
 Compressive strength, <i>f</i> <sub>c</sub> (MPa)	53.09	46.08
Young's modulus, $E_c$ (MPa)	34.7 × 10 <sup>3</sup>	31.6 × 10 <sup>3</sup>
Poisson's ratio, v	0.22	0.22
Direct tensile strength, $f_{td}$ (MPa)	3.77	3.66
Indirect tensile strength, $f_{ti}$ (MPa)	4.84	4.35
Fracture energy, $G_F(N/mm)$	0.137	0.137

The used numerical model is very sensitive to the length of the loading steps. Figure 7 shows the load/CMOD curve of the IISC-Y beam modeled with three displacement steps (10<sup>-2</sup>, 10<sup>-3</sup> and 10<sup>-4</sup> mm). The improvement is significant while the loading step decreased, especially at the maximum loading point. The post-cracking part is almost the same for the three settings. A loading step of 10<sup>-4</sup> mm is set for the three mixes, seeking a balance between accuracy and computational cost.

INPUT	IISC-M	IISC-Y
Young's modulus (MPa)	210 × 10 <sup>3</sup>	6 × 10 <sup>3</sup>
Poisson's ratio	0	0
Tensile strength (MPa)	1,200	400
Fiber length/diameter (mm/mm <sup>2</sup> )	35/0.55	35/0.93
Fiber volume content (%)	0.5	0.5
Fiber shape	Hooked-end	Dimpled-surface

 $\begin{array}{c} 3 \\ 3 \\ 2.5 \\ 1 \\ 2 \\ 1.5 \\ 0 \\ 0 \\ 1 \\ 2 \\ 1.5 \\ 0 \\ 0 \\ 1 \\ 2 \\ 2 \\ 1.5 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \\ 2 \\ 3 \\ 4 \\ 5 \\ CMOD (mm) \end{array}$ 

# Figure 7: Comparison of IISC-Y notched beams modeled with different pseudo-step length.

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373 *4.2. Validation of the Finite Element Model* 

The structural responses of the three samples are presented in Figure 8 in terms of load/CMOD. The numerical results are computed with the tensile strength from both the Dog-bone test ( $f_{td}$ ) and the Brazilian test ( $f_{ti}$ ) as inputs.

377 The numerical curves are in good agreement with the experimental ones in

both tests, although there are slight differences between them.

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Figure 8: CMOD/load curve of IISC-M and IISC-Y notched beams.

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Focusing on the first stage (pre-cracking) of the curves, the mechanical behavior is mainly described by the tensile strength and the elastic parameters of the mixes. It is evident that the curves that are modeled using  $f_{td}$  shows a better fit than the models that are defined using  $f_{ti}$ . At the peak, the difference of the curves modeled with  $f_{td}$  and  $f_{ti}$  compared to the experimental curve for IISC-M is 0.5 % and 13.5 %, respectively, while for IISC-Y is 45 %
and 63 %. One explanation is that the interface elements are only damaged by
normal stresses at the base of the interface element and not by shear stresses.
The Dog-bone test represents this fracture mode (Mode I) better than the
Brazilian test. Even though the Brazilian test is standardized and commonly
used to define tensile strength, the results confirmed that the Dog-bone test
provides more suitable inputs.

394 The post-cracking stage mainly depends on the fiber properties and 395 quantity (bridging phenomena). The parameters of the bridging law 396 presented in Equation 5 are defined as  $t_1 = 0.6$ ,  $t_2 = 1.45$ ,  $c_1 = 1.7$ , and  $c_2 = 2.0$ 397 for IISC-M and as  $t_1 = 0.1$ ,  $t_2 = 0.2$ ,  $c_1 = 1.0$ , and  $c_2 = 0.6$  for IISC-Y. In Figure 8(a), 398 IISCM shows a hardening behavior in its post-cracking behavior while mix 399 IISC-Y shows a softening behavior, due to the fiber material difference. After 400 the first peak in IICS-M, there is a decline, reflecting the inactivity of the steel 401 fibers in response to concrete cracking. Then, the steel fibers are activated 402 and the better anchoring conditions meant that the load can be increased 403 after cracking. The improved anchoring effect is a result of the fiber shape 404 (hooked-end fibers) and the high elastic modulus, which increases the pull-405 out resistance. The numerical solution captures this effect and it remains 406 within the shaded range throughout the post-cracking stage. In Figure 8(b), 407 although the peak of the numerical solution overestimates the peak, the 408 proposed numerical model could also be considered suitable for synthetic 409 fibers.

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#### 411 *4.3. Numerical analysis*

Once the FEM is validated with the experimental load/CMOD curves, it
constitutes an advanced numerical tool that provides quantitative
information on every point of the beam at any loading step. It facilitates the

analysis of aspects that are difficult to estimate analytically, such as stress
field, internal displacements, crack development, damage level, and fiber
effectiveness.

418 Stresses are the responsible for crack formation and the numerical analysis provides the stresses of every element. Figure 9 illustrates the stress 419 420 concentration points and their evolution during the loading process. 421 Maximum stresses are located at the supports, loading point and the peak of 422 the crack. The crack reduces the load-bearing surface, which increases the 423 stresses throughout the beam and specially at the loading point. Once the 424 crack crosses the beam, the stress along the beam decreases while the CMOD 425 rapidly increases up until failure.



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Figure 9: Stress concentration at the peak of the crack (IISC-Y).

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430 As result of increasing stresses, the beam is damaged and cracks starts to 431 develop. The IISC notched beams shows brittle failure (sudden fracture) 432 while the mixes with fibers retained a residual strength, as may be concluded 433 from the tests (Figure 10). Both failure modes, as illustrated in Figure 11, 434 could be described through the interface elements. The load/deflection 435 curves computed through FEM are consistent with the failure modes observed in the experimentation. The IISC beams lose all their strength and 436 437 the crack crosses the beam at a deflection of 1 mm. Nevertheless, the fibers

- 438 sew the crack, which means that the reinforced mixes retained a residual
- 439 strength.
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Figure 10: Fracture mechanics

Figure 11: Load/deflection curve defined in FEM

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443 444

446 Table 5 presents the numerical results to assess the performance of 447 synthetic and steel fibers at deflections of 0.001, 0.004, 0.2, 1.0, and 3.5 mm. As they are analyzed at the same deflection, damage levels and CMOD are very 448 449 similar in both mixes. However, there are differences between the computed 450 load and the fracture energy that is applied to arrive at these states. The 451 difference is at times appreciable, especially after the deflection of 0.2 mm. 452 IISC-Y in particular loses most of its strength at a deflection of 1 mm where 453 the energy dissipation rate drops and crack openings begin to increase 454 significantly. At this point, fracture energy also starts to increase, in a way that
455 is consistent with the definition given in [57]. Mixes with steel fibers also
456 replicate this tendency. The computed fracture energy for both cases match
457 the experimental values presented in [32].

458

Table 5: Cracking development based on deflection (0.001, 0.004, 0.2, 1 and3.5 mm).

	Abs	solute Da	mage (%	)	
IISC-M	0.02	1.35	8.60	9.25	9.37
IISC-Y	0.01	1.03	9.12	9.35	9.36
		CMOD	(mm)		
IISC-M	0.003	0.012	0.161	1.006	3.765
IISC-Y	0.003	0.012	0.181	1.072	3.797
	Frac	ture enei	rgy (N/m	m)	
IISC-M	$7 \times 10^{-4}$	0.011	0.140	0.794	2.418
IISC-Y	$6 \times 10^{-4}$	0.001	0.110	0.243	0.688
Load (kN)					
IISC-M	2.2	9.7	13.8	17.2	8.1
IISC-Y	2.2	9.2	6.9	2.9	3.7

461

Table 6 depicts data to analyze the post-cracking performance according to fracture energy. It is worth noting that fracture energy is related to the crack growth resistance [57]. As the pull-out resistance of a synthetic fiber is lower, damage to IISC-Y appears earlier than in the mix with steel fibers. IISCM requires higher load values and shows a lower CMOD at the same energy 467levels. This substantial difference reveals the anchoring effect and higher468elastic modulus of the steel fibers. The corresponding loading points of the  $G_F$ 469under study are labeled in the load/deflection curves shown in Figure 11,470highlighting the effects of added fibers. The fracture energy that is required471by IISC-Y is dissipated by IISC-M at a deflection of 0.881 mm, underlining the472advantageous effects of steel fibers.

473

Table 6: Cracking development based on fracture energy (0.1, 0.182, 0.4, 0.688 and 2.418 N/mm)

Ŧ/J	0.000, and 2.410 N/mmj.

Damaged interface elements					
IISC-M	1,693	1,785	1,811	1,816	1,831
IISC-Y	1,822	1,861	1,861	1,861	-
	Relative damage level (%)				
IISC-M	87.74	93.92	97.46	98.57	100
IISC-Y	95.98	99.90	99.99	100	-
	Load (kN)				
IISC-M	15.20	13.40	15.33	16.56	8.23
IISC-Y	8.39	2.66	3.38	3.41	-
CMOD (mm)					
IISC-M	0.104	0.223	0.521	0.877	3.763
IISC-Y	0.147	0.617	2.099	3.795	-

476

Fracture mechanics may be studied in-depth through the behavior of the
interface elements. These are only introduced in the central area where
cracks are expected, due to the existence of a weakness (notch). Figure 12

480 shows crack growth and damage evolution on interface elements at different 481 values of *G<sub>F</sub>* (0.100, 0.182, 0.400, 0.688, and 2.418 N/mm). Damage level 482 starts to be relevant in the mix IISC-M about  $G_F = 0.001$  N/mm. Until then, 483 most of the interface elements are in the elastic domain.  $G_F = 0.182$  N/mm is 484 related to the activation point of the steel fibers while  $G_F = 0.400$  N/mm is an 485 intermediate point of the hardening curve of IISC-M.  $G_F = 0.688$  and 2.418 486 N/mm are fracture energy values at the failure points of IISC-Y and IISC-M, 487 respectively. The differences between the mixes are visible since the 488 beginning of the post-cracking stage. The crack crosses the IISC-Y beam at  $G_F$ 489 of 0.182N/mm whereas the IISC-M beam is at 0.400 N/mm. Another 490 interesting fact is damage distribution, specially at the top part of the beam. 491 Interface elements are slightly damaged, except for the crack in IISC-Y. 492 However, the damage to mix IISC-M is further distributed, showing higher 493 levels of damage in the top part. This fact is further evidence of the greater 494 ductility of the mixes reinforced with steel fibers.

The numerical results of this method confirmed that it is a promising tool for the fracture analysis of EAFS concrete. Not only may it be used for mixes reinforced with steel fibers [34,35], but it can also be used for synthetic fibers. The application of this technique can likewise provide quantitative information related to fracture mechanics that is difficult to determine through experimentation.





502 503

Figure 12: Development of damage level according to fracture energy (N/mm).

#### 505 **5. Conclusions**

A framework is presented in this paper for the study of fiber-reinforced EAFS concrete from an experimental and numerical point of view. A set of tests is proposed to define the inputs required by the numerical model, which is validated with notched beams made of fiber-reinforced EAFS concrete.

From a mechanical point of view, fibers influence mainly the tensile behavior (Fracture energy, toughness and residual stress) of the mixes changing the failure mode from brittle behavior to a more ductile behavior. In particular, the positive effect of steel fibers must be reported improving remarkably the postpeak behavior.

The numerical analysis shows that the proposed method is able to model fracture and fiber bridging effects. The tensile strength required by the bridging model is determined through direct and indirect tests. It is concluded that the proposed Dog-bone test provides more suitable values to describe the bridging model implemented on the interface elements.

520 The numerical framework provides valuable information to understand 521 damage and cracking mechanisms. The numerical results confirms the validity for synthetic fibers and provides further evidence that the interfaceelements are suitable for modeling the steel-fiber bridging effect.

524 The framework that has been developed can facilitate deeper 525 understanding of the effects of fibers at a material-scale, through the study of 526 fiber materials, volume, content, and shape. It is also a promising tool to apply 527 to large-scale structures.

528 Importantly, future research could well investigate the behavior of EAFS 529 concrete in large-scale structures under varied loading conditions. In 530 addition, further research is needed to confirm the good performance of the 531 numerical method at producing accurate descriptions of the fiber bridging 532 effect.

533

## 534 Acknowledgment

535 The authors wish to express their gratitude to the following entities for 536 having funded this research work: the Spanish Ministries MCI, AEI, EU and 537 ERDF [RTI2018-097079-B-C31; PID2020-113837RB-I00; 538 10.13039/501100011033]; the Junta de Castilla y León (Regional Government) and ERDF [UIC-231, BU119P17]; the Basque Government 539 540 research group [IT1619-22 SAREN]; the University of Burgos [Y135.GI]. 541 Likewise, our thanks to CHRYSO and HORMOR for supplying the materials 542 used in this research.

543

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