Transition Probability Matrices for pavement deterioration modelling with variable duty cycle times

Ángela Alonso-Solorzano[a], Heriberto Pérez-Acebo[b]*, Daniel J. Findley[c], and Hernán Gonzalo-Orden[d]

[a]*Department of Pyshics, University Francisco de Vitoria, Madrid, Spain*

[b]*Mechanical Engineering Department, University of the Basque Country UPV/EHU, Bilbao, Spain*

[c]*Institute for Transportation and Research Education, North Carolina State University, Raleigh, NC, USA.*

[d]*Department of Civil Engineering, University of Burgos, Burgos, Spain;*

* Corresponding author: Heriberto Pérez-Acebo, Pº Rafael Moreno "Pitxitxi", 2, 48013, Bilbao, Spain, email: heriberto.perez@ehu.eus

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Probabilistic pavement models are generally considered to be able to capture an accurate representation of the performance of in situ pavements, with the Markov chains being the most widely used type. Homogeneous Markov chain models present the same transition probability matrix (TPM) for all the transitions of the analysed period and require data from multiple duty cycles (or step times) of one or two years. The aim of this paper is to explore the feasibility of developing homogeneous Markov chain models with variations of the step time (in increments of either one or two years). Without considering maintenance and rehabilitation (M&R) works, this research found that TPMs for a one-year duty cycle can be calculated from the two-year duty cycle, thereby reducing the frequency and cost of data collection without a noticeable effect on accuracy using International Roughness Index (IRI) values from the Spanish State Road Network. However, for developing coherent TPMs, two primary assumptions were made: 1) heavy vehicle traffic volumes determine the road segment classifications, and 2) only roads from the same climatic region were modelled. Additional assumptions were developed to account for: 3) M&R activities and IRI seasonal variations or measurement errors, and 4) the initial year value was used if multiple values existed for the traffic category (TC). Results showed that TPMs in the same road for a TC were relatively similar when using a one-year step time. A typical maximum range of 0.10-0.15 between maximum and minimum values was found when comparing TPMs across the same TC. This variability comes from the inherent stochastic nature of pavements. The satisfactory results verified the validity of the methodology and overcome the disadvantages of homogeneous Markov models. Furthermore, the results suggest that pavement sections are adequately designed in Spain for each TC because of the similar deterioration patterns.

Keywords: pavement performance model; probabilistic model; homogeneous Markov chain; International Roughness Index; pavement management system;

Introduction

The use of pavement management systems (PMS) is an indispensable element for highway administrations to optimize the limited available budget for road maintenance and rehabilitation (M&R). PMS have been defined as a set of tools or methods that assist decision makers in finding optimum strategies for providing and maintaining pavements in a serviceable condition over a given period of time (Hudson *et al.* 1979, AASHTO 1993, Haas *et al*. 1994,). In other words, PMS are a systematic approach that provides quantifiable engineering information to help administrators and engineers manage road pavements (Uddin 2006). Their function is to improve the efficiency of decision-making, expand the scope, provide feedback on the consequences of various decisions, facilitate the coordination of activities within the road agency, and ensure the consistency of decisions made at different management levels within the same organization (Haas *et al*. 1994). The main elements composing a PMS are (Bandara and Gimaratne, 2001, AASHTO 2012): 1) Pavement data collection for present condition evaluation, 2) Prediction of the future pavement condition, conducted by means of pavement performance models, and 3) Network and project-level maintenance and rehabilitation (M&R) plans taking into consideration local traffic and conditions, along with material and financial resources.

Regarding the pavement condition evaluation, one or more indices may be used, which are generally obtained directly from measurements on the surface of the road (Shahnazari *et al*. 2012, Soncim *et al*. 2018). Additionally, there exist many categories for grouping the condition data. For example, the Pavement Management Guide (AASHTO 2012) classifies them into surface characteristics (including roughness, surface texture, and friction), pavement distresses, subsurface characteristics, and structural evaluation. Nevertheless, every road administration selects the parameters or indices to employ and, hence, there is not a global consensus about the most important properties to measure (Flintsch and McGhee 2009).

Although surface characteristics represent only a small proportion of the total pavement structure, they are a vital item because they are the only point of contact between the vehicle and the infrastructure. Among existing surface parameters, roughness is highlighted as essential. Pavement roughness, defined as the deviation of a pavement surface from a true planar surface with characteristic dimensions that affect vehicle dynamics, ride quality, dynamic loads and pavement drainage (ASTM 2020), is the attribute of most interest to users because road users perceive it as the means of a better or worse riding experience (Smith and Ram 2016). Studies showed that roughness is the primary criteria for users to rate the pavement's performance, and hence, the road network condition (Budras 2001). Additionally, there is a proven direct relationship between pavement roughness and vehicle operating costs, due to its incidence on the rolling resistance, increasing fuel consumption, and vehicle maintenance costs (Smith *et al*. 1997, Chatti and Zaabar 2012, Zang *et al*. 2018). Consequently, the FHWA considered roughness as the single most impact factor affecting riding quality (Smith and Ram 2016) and it can represent the presence of major surface distresses, such as potholes, cracking, ravelling, etc. (Mubaraki 2010). During the 1980s, the World Bank conducted the International Road Roughness Experiment (IRRE) with the aim of correlating the results from existing multiple roughness measurement devices (Sayers *et al*. 1986a). During the data processing, the research showed that all the roughness-measuring devices around the world could produce measures on the same scale, as long as that scale had been suitably selected. Consequently, the International Roughness Index (IRI) was developed (Sayers *et al*. 1986b). The IRI is a mathematical representation of the accumulated suspension stroke of a vehicle, divided by the distance travelled by the vehicle during the data collection process. It is based on a simulation roughness response of a standard quarter-car at a speed of 80 km/h and its usual units for IRI are m/km or mm/m (Sayers 1995). At present, due to its transferability around the world and its stability over time, it has become the most widely employed index for measuring roughness, with examples around the world, both in developed countries (Ziari *et al*. 2016, Mazari and Rodríguez 2016, Sidess *et al*. 2020, Pérez-Acebo *et al*. 2020, Yamany *et al*. 2020) and in developing countries (Alburqueque and Nuñez 2011, Sandra and Sarkar 2013, Pérez-Acebo *et al*. 2018, Gharieb *et al*. 2022).

With regard to the pavement performance models, also known as pavement deterioration models, evolution models, or pavement performance prediction models (Justo-Silva *et al*. 2021); they are able to predict the future condition, resulting in key elements in any advanced PMS. There are many performance model types that may be classified according to a variety of criteria. For example, the Pavement Management Guide (AASHTO 2012) classified them in deterministic models, probabilistic models, Bayesian models, and subjective (or expert-based) models. Haas et al. (1994) summarized the models into four basic types: purely mechanistic, mechanistic-empirical, regression (or deterministic), and subjective (which could include probabilistic models as they are developed subjectively sometimes). Uddin (2006), apart from deterministic and probabilistic models (where Bayesian and Markov chains models were included), indicated the Artificial Neural Network (ANN) models as another type. In fact, ANN models have received great attention in the last two decades (Pérez-Acebo *et al*. 2018, Justo-Silva *et al*. 2021). Nonetheless, deterministic and probabilistic models are generally recognized as the basic groups (Abaza 2016b, Amin and Amador-Jiménez 2016, Alaswadko and Hwayyis 2022, Fani *et al*. 2022, Pérez-

Acebo *et al*. 2022a). Although the ANN models can obtain greater accuracy than the other models (Abdelaziz *et al*. 2020; Yamany *et al*. 2020), they are regarded as a "*black box*" by many authors, as they do not publicly show the relationship between the predicted variable(s) and its/their predictors (Gurney 1997, Sollazo *et al*. 2017, Garcia de Soto *et al*. 2018, Pérez-Acebo et al. 2022a).

Deterministic models establish a relationship between a predicted variable or variables, the dependent variable(s); and the predicting variable(s), the independent variable(s). Generally, regression analysis is conducted for developing these models with a goal of finding the best static fit of the data. A usual approach in pavement management is the least square regression techniques, minimizing the sum of the squared differences between the developed curve and the available data points. Various curves can be used such linear, quadratic, sigmoid, etc. On the contrary, probabilistic models provide the probabilistic distribution of the predicted variable, not a precise value. Thus, apart from predicting the future condition of the pavement, as deterministic models do; probabilistic, or stochastic, models introduce uncertainty in pavement deterioration modelling, which is said to be closer to reality (Golroo and Tighe 2009, Thomas and Sobanjo 2012). Pavement evolution is recognized to be probabilistic in nature and, hence, assuming some level of uncertainty is more realistic (Li *et al*. 1997, Tjan and Pitaloka, 2005, Hong 2014; Abaza 2016a, 2016b).

Deterministic models are frequently used by highway administrations with large historical pavement condition information, whereas probabilistic ones can be created when little information is available. Stochastic models can be developed even when there is only information available from two data collection periods (Mohammadi *et al*. 2019).

Among probabilistic models, the Markov chains are the most widely used (Ortiz-García *et al*. 2006; Adedimila *et al*. 2009, Soncim *et al*. 2017; Yamany *et al*. 2019, 2021). To build these models, the necessary inputs are condition states, different categories for the predicted variable, and duty cycles (or step times). Then, matrices are generated and the probability that a section in a category at a given stage will remain in the same category (condition state) or jump to another category in the next stage/period is forecasted. The time period is the interval between consecutive dates of data collection. The most common value reported in the literature is one year because most road agencies collect pavement condition on an annual basis and all types of models are proposed with this frequency. Moreover, it is also possible to use pavement data collected with variable frequency for developing deterministic and probabilistic models (Alaswadko 2016, Alaswadko et al. 2018, 2019, Pérez-Acebo et al. 2020, Hwayyis et al. 2022). Additionally, Alaswadko and Hwayyis (2022) analysed the supplementary inconsistency between time series and concluded that it is necessary to consider the existing uncertainty in some parameters and captures the effect of observed and unobserved parameters, resulting in a more accurate model.

With regard to Markov chains, one year is also the usual duty cycle (Butt *et al*. 1987, Abaza 2016a), but a duty cycle of 2 years (Hassan *et al*. 2017a, 2017b) and of six-month have been adopted (Pérez-Acebo et al. 2019). However, Markov chains have not been developed from data with variable duty cycles, i.e., from data where the frequency of data collection is not constant.

Hence, the objective of this paper is to explore the feasibility of developing a Markov Chain model with data collection consisting of a variable time frequency, i.e., with data separated one year and two years, and if so, what assumptions are necessary. With this aim, IRI data were obtained from the Ministry of Transports, Mobility and

Urban Agenda (MTMUA) of Spain, collected during different years (2004, 2005, 2006, 2007, 2009 and 2011). Overall, this paper demonstrates that accurate matrices can be obtained even if various duty cycles are introduced in the pavement performance modelling.

Section 2 presents the theoretical framework for the transition probability matrices and how they can be developed with different time steps. Available data, the processing method, and the assumed conditions are shown in Section 3. In section 4, the results are discussed. Finally, conclusions are listed in section 5.

Theory about transition probability matrices and development of matrices for variable duty cycle times

Theoretical aspects about transition probability matrices

The Markov model is a stochastic process ruled by the following three restrictions: 1) discrete in time, 2) countable or finite state space and, 3) satisfaction of the 'Markov property' (Isaacson and Madsen 1976). The Markov property states that any future condition state of the process is only dependent on its present state and not on the past states (Hillier and Leberman 1990). It is generally assumed that pavement deterioration fulfils the Markov property (Kerali and Snaith 1992). A Markov model has three main features: 1) the condition state, 2) the step cycle, and 3) the transition probability matrix.

The condition states (CS) are the discrete ratings assigned to the infrastructure condition. They can be categorical, such as excellent, fair, and poor (Yamany *et al.* 2021), or using discrete values of a continuous variable (from 100 to 90.1, from 90 to 80.1, etc.). Associated with the condition state, the state probability vector, or condition probability vector, appears as a row vector, $A = \{a_1, a_2\ldots, a_i\ldots, a_n\}$, showing the current condition of the infrastructure by means of the proportions of it in each condition state.

For example, if we have 5 CS, and 30% of the sections are in CS 1, 20% in CS 2, 17% in CS 3, 20% in CS 4, and 13% in CS 5, Equation (1) shows the vector:

$$A = \{0.30, 0.20, 0.17, 0.20, 0.13\} \tag{1}$$

The sum of all the elements, a_i, must be equal to one and all the elements must be nonnegative.

The duty cycle, or step time, is the time interval between two stages. The most common frequency of data collection is one year (Ortiz-García *et al*. 2006, Yamany *et al*. 2021).

The transition probability matrix (TPM), usually denoted as P, represents the deterioration with time. It is a squared matrix, with *n* rows and *n* columns; where *n* is the number of condition states (CS) considered. Equation (2) shows its general form.

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix} \tag{2}$$

where each element, p_{ij}, indicates the probabilities of an element in CS *i* in stage *t* to change to condition state *j* in stage *t* + 1, as indicated in Equation (3).

$$p_{ij} = prob[X(t+1) = j / X(t) = i] \tag{3}$$

In other words, p_{ij} can be regarded as the conditional probability of any parameter (the employed variable) in state *i* in the current stage that will be in state *j* after just one cycle time. Any element of the TPM must follow the following two restrictions, Equation (4) and (5) (Wang *et al*. 1994):

$$0 \le p_{ij} \le 1, \text{ for all } i \text{ and } j, \text{ and } i, j = 1, 2, \ldots, n \tag{4}$$

$$\sum_{j}^{n} p_{ij} = 1, \text{ for each } i \text{ and } j = 1, 2, \ldots, n \tag{5}$$

The elements of the main diagonal, p_{ii}, within the TPM, indicate the probability of a section to remain in the same condition (in the same CS) after one duty cycle. Additionally, elements p_{ij}, where $i > j$, refer to sections that evolve to a better state after the step time. This is impossible unless a maintenance or rehabilitation (M&R) activity was conducted during the time period because pavement performance cannot improve its state without intervention. Hence, if we follow normal pavement deterioration processes and assumptions (i.e., assuming that these instances are due to measurement errors), elements with i greater than j must be equal to 0. Similarly, a pavement section in the worst condition category cannot further deteriorate. Moreover, as a consequence of this idea and Equation (5), element p_{nn} must be equal to 1. Therefore, the general form of a matrix representing a typical deterioration process has the form of Equation (6)

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1n} \\ 0 & p_{22} & p_{23} & \cdots & p_{2n} \\ 0 & 0 & p_{33} & \cdots & p_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \ldots & 1 \end{bmatrix} \tag{6}$$

Once the TPM is calculated, the condition vector of the next state is calculated by multiplying the present state vector and the TPM, as presented in Equation (7) and (8):

$$A_1 = A_0 \times P_1 \tag{7}$$

$$A_2 = A_1 \times P_2 \tag{8}$$

Where A_0, A_1, and A_2 are the condition vector of the road at time $t = 0$, 1, and 2, respectively; and P_1 and P_2 are the TPMs corresponding to transitions from time $t = 0$ to 1 and from $t = 1$ to 2, respectively.

Markovian models for predicting pavement performance can be classified into homogeneous Markov, staged-homogeneous Markov, nonhomogeneous Markov, semi-Markov, and hidden Markov models (Yamany and Abraham 2021). The accuracy of the models depends on the selected type of Markovian models, explanatory variables, and data quality and availability (Yamany and Abraham 2021).

Homogeneous Markov models are not time dependent, i.e., the TPM is constant over time. Hence, the same matrix can be used to any t to $t + 1$ transition, assuming that the deterioration rate over the analysis period does not change (Pérez-Acebo *et al*. 2019). Therefore, condition state vectors in any required future time *t,* or after t transitions can be calculated using Equation (9) t times, following Champman-Kolmogorov equations:

$$A_t = A_{t-1} \times P = A_{t-2} \times P \times P = A_{t-2} \times P^2 = A_{t-3} \times P^3 = \cdots = A_0 \times P^t \quad (9)$$

For developing this type of model, a limited number of transitions are needed; and it is even possible to create a model with only two transitions (Mohammadi *et al*. 2019). As a result, many authors have selected this methodology (Kulkarni 1984, Carnahan, 1988, MacLeod and Walsh 1998, Chou *et al*. 2008, Pulugurta *et al*. 2009, Abaza and Murad 2010, Perez-Acebo *et al*. 2018). However, as a trade-off for this simplicity, homogeneous Markov models are not as accurate as other models because of the assumption that pavement deterioration is stationary, which is contrary to the continuous natural change in pavement evolution rates over time (Butt *et al*. 1987, Abaza 2016b). Traffic loads, environmental factors, or subgrade strength may change during pavement life and could lead to different pavement deterioration progression (Li *et al*. 1997, Hong and Wang, 2003).

Staged-homogeneous Markov models, presented by Butt *et al*. (1987), assume that the TPMs do not change significantly over a stage or time interval of approximately 5 or 6 years. Hence, this methodology requires the development of multiple constant homogenous TPMs, one for each time interval. Therefore, this type of model is more reliable than homogeneous models for predicting future pavement conditions but they are not able to capture changes in pavement evolution within the defined stages. Additionally, staged-homogeneous models require more data than homogeneous models.

Semi-Markov models suppose that transition probabilities vary according to the change in pavement deterioration over uneven intervals, called holding times. These models are more accurate than the previously described models because they do not assume stationary transition probabilities and consider changes in pavement evolution rate at unequal stages. Nevertheless, more pavement condition information is necessary to calculate the uneven length of the intervals and are not adequate when insufficient data are available (Thomas and Sobanjo 2013). In Nonhomogeneous Markov models, the TPM changes over the analysis period for each transition. In other words, this type of model better fits the random performance of pavements compared with previous models (Madanat *et al*. 1995, Yang *et al*. 2005, 2006, Kobayashi *et al*. 2010, Tabatabaee and Ziyadi 2013). Nevertheless, they require a large quantity of data and more computation time and, hence, road agencies prefer not to develop nonhomogeneous models until enough data are available. Finally, hidden Markov models suppose two condition state types: observed and hidden. For observed condition states, there are pavement distresses that can be directly measured, such as potholes, cracks, etc. For unobservable or hidden condition states, there are pavement condition indexes, such as IRI or Pavement Condition Index (PCI). The TPMs of hidden states, such as IRI, can be calculated using data from field observations and measurements. Hidden Markov models may help in establishing the relationship between observed and hidden states; they are used when pavement condition information is incomplete (Lethanh and Adey 2012, 2013).

Additionally, five methods are proposed in the literature to estimate the TPMs for pavement performance: expected-value, percentage transition, simulation-based, econometric models, and duration models (Ortíz-García *et al*. 2006, Abaza 2016b, Yamany *et al*. 2019a). Among them, the percentage transition is the most widely used and it was employed in this research. This method estimates the transition probabilities as the proportion of pavement sections that transition from one condition state to another after one step time. The proportions of a fixed length or the cumulative length of pavement sections of varying lengths can be used (Hassan *et al*. 2017a, Osorio-Lird *et al*. 2018, Pérez-Acebo *et al*. 2018). It is calculated from Equation (10):

$$p_{ij} = \frac{N_{ij}}{Ni} \tag{10}$$

Where p_{ij} is the matrix element in row i and column j, N_{ij} is the number of sections that shift from state i to state j during one step time, and N_i is the total number of sections that were in condition i before the transition.

As commented in section 1, the length of duty cycle is the time interval between consecutive dates of data collection (Yamany *et al*. 2021). Generally, the most widely employed step time for Markov chains is one year because most highway administrations collect pavement condition data on an annual basis (Abaza 2016a, 2016b), but examples of two years for the step time (Hassan *et al*. 2015) and even half a year (Pérez-Acebo *et al*. 2019) can be found. Yamany *et al*. 2021 analysed the importance of using one or two years as the length of the duty cycle, providing useful results for different combinations of a number of condition states, length of the duty cycle, and stage length (for a staged-homogenous Markov model) depending on the selected methodology (homogeneous Markov, staged-homogeneous, and semi-Markov models). However, there are currently no results from investigations of the feasibility for using data from variable length of duty cycles, i.e., when data collection frequency can be one and two years.

Consequently, homogeneous Markov models with data from inconsistent data collection frequency will be developed. Homogeneous Markov models were observed as the most suitable type to combine the variable duty cycles. Nonetheless, the disadvantages of the homogeneous Markov models were accounted for based on relevant assumptions and hypotheses, improving the accuracy of the models.

In next subsection, the mathematical procedure to combine data from variable lengths of duty cycle is explained.

Matrices for variable cycle times

As shown in Equation (9), in homogeneous Markov models, the same matrix can be employed during the entire analysed period. If we have a time step of one year, the transition probability matrix, P, satisfies Equation (11), and for data with a difference of two years, the TPM, Q in this case, satisfies Equation (12).

$$A_{t+1} = A_t \times P \tag{11}$$

$$A_{t+2} = A_t \times Q \tag{12}$$

In a homogeneous Markov model, the transitions are assumed to be similar during all the pavement life, and hence, Equation (11) and (12) are combined into Equation (13):

$$A_{t+2} = A_{t+1} \times P = A_t \times P \times P = A_t \times P^2 = A_t \times Q \tag{13}$$

It results in Equation (14):

$$\mathrm{P}^2 = \mathrm{P} \times \mathrm{P} = \mathrm{Q} \tag{14}$$

Consequently, after matrix *Q*, which corresponds to a step time of two years, is obtained; it is necessary to calculate a matrix *P*, which is multiplied by its own results.

TPMs are squared matrixes, *n* x *n*, so each one has n^2 elements. There are n^2 unknown values (the elements of matrix P, elements p_{ij}) and there are n^2 equations. Unique solutions are not always possible because the resulting equations are not linear (some variables are quadratic) and solving this system of equations can be difficult. However, in pavement deterioration modelling, some simplifications help solving the calculation of *P*. If no improvements in condition state are assumed (which means that no M&R activities are conducted), as seen in Equation (6), elements below the main diagonal (p_{ij} with $i > j$) are equal to zero, and p_{nn} is equal to 1. Hence, Equation (14) can be written as Equation (15):

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1n} \\ 0 & p_{22} & p_{23} & \cdots & p_{2n} \\ 0 & 0 & p_{33} & \cdots & p_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} \times \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1n} \\ 0 & p_{22} & p_{23} & \cdots & p_{2n} \\ 0 & 0 & p_{33} & \cdots & p_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} & q_{13} & \cdots & q_{1n} \\ 0 & q_{22} & q_{23} & \cdots & q_{2n} \\ 0 & 0 & q_{33} & \cdots & q_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} \tag{15}$$

There are $n + (n - 1) + (n - 2) + ...+ 1$ unknown variables (although the solution for element p_{nn} is known) and the same quantity of equations to solve it. For example, if we establish five CS, which is the recommended option for step times of one year for homogeneous Markov models (Yamany *et al*. 2021). Equation (15) results in Equation (16):

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} \\ 0 & p_{22} & p_{23} & p_{24} & p_{25} \\ 0 & 0 & p_{33} & p_{34} & p_{35} \\ 0 & 0 & 0 & p_{44} & p_{45} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} \\ 0 & p_{22} & p_{23} & p_{24} & p_{25} \\ 0 & 0 & p_{33} & p_{34} & p_{35} \\ 0 & 0 & 0 & p_{44} & p_{45} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} & q_{15} \\ 0 & q_{22} & q_{23} & q_{24} & q_{25} \\ 0 & 0 & q_{33} & q_{34} & q_{35} \\ 0 & 0 & 0 & q_{44} & q_{45} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (16)$$

From the matrix multiplication of Equation (16), 5 + 4 + 3 + 2 + 1 = 15 equations are obtained, presented in Equations (17)-(31):

$$q_{11} = p_{11} \cdot p_{11} = p_{11}^2 \quad (17)$$

$$\mathrm{q}_{12} = \mathrm{p}_{11} \cdot \mathrm{p}_{12} + \mathrm{p}_{12} \cdot \mathrm{p}_{22} = \mathrm{p}_{12} \cdot (\mathrm{p}_{11} + \mathrm{p}_{22}) \quad (18)$$

$$q_{13} = p_{11} \cdot p_{13} + p_{12} \cdot p_{23} + p_{13} \cdot p_{33} = p_{13} \cdot (p_{11} + p_{33}) + p_{12} \cdot p_{23} \quad (19)$$

$$q_{14} = p_{11} \cdot p_{14} + p_{12} \cdot p_{24} + p_{13} \cdot p_{34} + p_{14} \cdot p_{44} = p_{14} \cdot (p_{11} + p_{44}) + p_{12} \cdot p_{24} + p_{13} \cdot p_{34} \quad (20)$$

$$q_{15} = p_{11} \cdot p_{15} + p_{12} \cdot p_{25} + p_{13} \cdot p_{35} + p_{14} \cdot p_{45} + p_{15} \cdot p_{55=} = p_{15} \cdot (p_{11} + p_{55}) + p_{12} \cdot p_{25} + p_{13} \cdot p_{35} + p_{14} \cdot p_{45} \quad (21)$$

$$\mathrm{q}_{22} = \mathrm{p}_{22} \cdot \mathrm{p}_{22} = \mathrm{p}_{22}^2 \quad (22)$$

$$\mathrm{q}_{23} = \mathrm{p}_{22} \cdot \mathrm{p}_{23} + \mathrm{p}_{23} \cdot \mathrm{p}_{33} = \mathrm{p}_{23} \cdot (\mathrm{p}_{22} + \mathrm{p}_{33}) \quad (23)$$

$$\mathrm{q}_{24} = \mathrm{p}_{22} \cdot \mathrm{p}_{24} + \mathrm{p}_{23} \cdot \mathrm{p}_{34} + \mathrm{p}_{24} \cdot \mathrm{p}_{44} = \mathrm{p}_{24} \cdot (\mathrm{p}_{22} + \mathrm{p}_{44}) + \mathrm{p}_{23} \cdot \mathrm{p}_{34} \quad (24)$$

$$\mathrm{q}_{25} = \mathrm{p}_{22} \cdot \mathrm{p}_{25} + \mathrm{p}_{23} \cdot \mathrm{p}_{35} + \mathrm{p}_{24} \cdot \mathrm{p}_{45} + \mathrm{p}_{25} \cdot \mathrm{p}_{55} = \mathrm{p}_{25} \cdot (\mathrm{p}_{22} + \mathrm{p}_{55}) + \mathrm{p}_{23} \cdot \mathrm{p}_{35} + \mathrm{p}_{24} \cdot \mathrm{p}_{45} \quad (25)$$

$$q_{33} = p_{33} \cdot p_{33} = p_{33}^2 \tag{26}$$

$$q_{34} = p_{33} \cdot p_{34} + p_{34} \cdot p_{44} = p_{34} \cdot (p_{33} + p_{44}) \tag{27}$$

$$q_{35} = p_{33} \cdot p_{35} + p_{343} \cdot p_{45} + p_{354} \cdot p_{55} = p_{35} \cdot (p_{33} + p_{55}) + p_{34} \cdot p_{45} \tag{28}$$

$$q_{44} = p_{44} \cdot p_{44} = p_{44}^2 \tag{29}$$

$$q_{45} = p_{44} \cdot p_{45} + p_{45} \cdot p_{55} = p_{45} \cdot (p_{44} + p_{55}) \tag{30}$$

$$q_{55} = p_{55} \cdot p_{55} = p_{55}^2 \tag{31}$$

This system of equations has a unique solution. Elements in the main diagonal have a similar direct solution: $p_{11} = \sqrt{q_{11}}$; $p_{22} = \sqrt{q_{22}}$; $p_{33} = \sqrt{q_{33}}$; $p_{44} = \sqrt{q_{44}}$; $p_{55} = \sqrt{q_{55}}$, which are shown in Equation (32):

$$p_{ii} = \sqrt{q_{ii}} \tag{32}$$

The solution for element p_{12} is shown in Equation (33). Observing the rest of the elements of the form $p_{i,i+1}$, Equation (34) presents its solution.

$$p_{12} = \frac{q_{12}}{\sqrt{q_{11}}+\sqrt{q_{22}}} \tag{33}$$

$$p_{i,i+1} = \frac{q_{i,i+1}}{\sqrt{q_{ii}}+\sqrt{q_{i+1,i+1}}} \tag{34}$$

The solution of element p_{13} is given in Equation (35) and the general solution of the form $p_{i,i+2}$.in Equation (36).

$$p_{13} = \frac{q_{13}-\left(\frac{q_{12}}{\sqrt{q_{11}}+\sqrt{q_{22}}}\right)\cdot\left(\frac{q_{23}}{\sqrt{q_{22}}+\sqrt{q_{33}}}\right)}{\sqrt{q_{11}}+\sqrt{q_{33}}} \tag{35}$$

$$p_{i,i+2} = \frac{q_{i,i+2} - \left(\frac{q_{i,i+1}}{\sqrt{q_{ii}}+\sqrt{q_{i+1,i+1}}}\right)\cdot\left(\frac{q_{i+1,i+2}}{\sqrt{q_{i+1,i+1}}+\sqrt{q_{i+2,i+2}}}\right)}{\sqrt{q_{ii}}+\sqrt{q_{i+2,i+2}}} \tag{36}$$

Similarly, the solution for element p_{14} is presented in Equation (37) and the general form for elements $p_{i,i+3}$ is given in Equation (38).

$$p_{14} = \frac{q_{14} - \left(\frac{q_{12}}{\sqrt{q_{11}}+\sqrt{q_{22}}}\right)\cdot\frac{q_{24} - \left(\frac{q_{23}}{\sqrt{q_{22}}+\sqrt{q_{33}}}\right)\cdot\left(\frac{q_{34}}{\sqrt{q_{33}}+\sqrt{q_{44}}}\right)}{\sqrt{q_{22}}+\sqrt{q_{44}}} - \left(\frac{q_{13} - \left(\frac{q_{12}}{\sqrt{q_{11}}+\sqrt{q_{22}}}\right)\cdot\left(\frac{q_{23}}{\sqrt{q_{22}}+\sqrt{q_{33}}}\right)}{\sqrt{q_{11}}+\sqrt{q_{33}}}\right)\cdot\left(\frac{q_{23}}{\sqrt{q_{22}}+\sqrt{q_{33}}}\right)}{\sqrt{q_{11}}+\sqrt{q_{44}}} \tag{37}$$

$$p_{i,i+3} = \frac{q_{i,i+3} - \left(\frac{q_{i,i+1}}{\sqrt{q_{ii}}+\sqrt{q_{i+1,i+1}}}\right)\cdot\frac{q_{i+1,1+3} - \left(\frac{q_{i+1,i+2}}{\sqrt{q_{i+1,1+1}}+\sqrt{q_{i+2,i+2}}}\right)\cdot\left(\frac{q_{i+2,i+3}}{\sqrt{q_{i+2,i+2}}+\sqrt{q_{i+3,i+3}}}\right)}{\sqrt{q_{i+1,i+1}}+\sqrt{q_{i+3,i+3}}} - \left(\frac{q_{i,i+2} - \left(\frac{q_{i,i+1}}{\sqrt{q_{ii}}+\sqrt{q_{i+1,i+1}}}\right)\cdot\left(\frac{q_{i+1,i+2}}{\sqrt{q_{i+1,i+1}}+\sqrt{q_{i+2,i+2}}}\right)}{\sqrt{q_{ii}}+\sqrt{q_{i+2,i+2}}}\right)\cdot\left(\frac{q_{i+1,i+2}}{\sqrt{q_{i+1,i+1}}+\sqrt{q_{i+2,i+2}}}\right)}{\sqrt{q_{ii}}+\sqrt{q_{i+3,i+3}}} \tag{38}$$

Element p_{15}, and in general, elements of the form $p_{i,i+4}$ can be assumed to be equal to 0 in most of the cases. Many authors indicate that, without M&R activities, road sections can only remain in the same condition or deteriorate to the next condition, implying that only element p_{ii} and $p_{i,i+1}$ can appear in the TPM (Butt *et al*. 1987, Abaza 2004, Ortiz-García *et al*. 2006). However, Pérez-Acebo *et al*. (2018, 2019) showed that it is better not to assume this concept in advance. Results showed that elements $p_{i,i+2}$ and $p_{i,i+3}$ were not equal to zero and in some cases, even $p_{i,i+4}$ was also not equal to zero. The general solution for this element is given in Equation (39). To avoid an unnecessarily large expression, some values are expressed as values of the *P* matrix, which must be previously obtained, by means of Equations (34), (36), and (38).

$$p_{i,i+4} = \frac{q_{i,i+4} - p_{i,i+1}\cdot p_{i+1,i+4} - p_{i,i+2}\cdot p_{i+2,i} - p_{i,i+3}\cdot p_{i+3,i+4}}{\sqrt{q_{ii}}+\sqrt{q_{i+4,i+4}}} \tag{39}$$

As shown, if the Q matrix for a duty cycle of two years is obtained, the P matrix for transitions of one year can be calculated. This methodology was employed to calculate the TPM with variable step times (one and two years).

Research methodology

Data collection

The Ministry of Transports, Mobility and Urban Agenda (MTMUA) of Spain is the main road agency in Spain and it is the owner and manager of the main road network in the country, called Red de Carreteras del Estado, which literally means State Road Network (SRN) (BOE 2015). Although the road network managed by the MTMUA represents less than 20% of the total interurban road network, more than 50% of the interurban mobility is conducted through this network, fact that is repeated in most of countries (Perez-Acebo *et al*. 2022b). In 2021, the total length of the SRN was 26,459.3 km, including 11,684.7 km of freeways, 479.5 km of multilane highways and 14,295 km of single carriageway roads (with 2 directions) (MTMAU 2022).

The Ministry pavement condition data of the SRN were published online and are accessible to anyone. It includes IRI data, cracks, and deflection values. Data can be filtered by road name (code), province, year, and initial and final kilometre points (KP). In the case of IRI, data are shown every 100 m, which is the standard length for roughness analysis in Spain (MFOM 2014), as in many other countries (Mucka 2017). A deeper look at the available data shows that for most of the two direction single carriageway roads (one lane per direction, in a unique carriageway), data are only given for the following years: 2004 or 2005, 2006, 2007, 2009, and 2011. As shown, data collection years are not consecutive and the frequency is not constant, ranging between one and two years. Consequently, the methodology presented in section 2 was applied to demonstrate that it was prudent to include all of the available data even when the data collection frequency is not constant.

Nevertheless, although the IRI evolution of each 100 m-long section is known, additional important information was not available. For example, the pavement age, the pavement structure, or years since the pavement was opened to traffic or since the last major rehabilitation are not provided. Therefore, a deterministic model cannot be applied due to the lack of information that is needed for this type of model (Abdelaziz *et al*. 2020, Pérez-Acebo *et al*. 2021). Hence, a probabilistic model, such as the Markov Chain is the most appropriate solution considering the available data.

It is generally known that road pavements deteriorate due to traffic loads, environmental factors, and construction deficiencies (Salas *et al*. 2018. Salas and Perez-Acebo 2018, Jiang *et al*. 2020, Sidess *et al*. 2021, Wen *et al*. 2022). Regarding traffic loads, the annual traffic volumes on each road section of the SRN are measured and can be found publicly on the webpage of the Ministry. In Spain, eight traffic categories (TC) are established according to the Annual Average Daily Traffic of Heavy Vehicles (AADTHV) in the project lane, which is the lane with the highest quantity of heavy vehicles on the road (hv/day) (Table 1). In Spain, a vehicle weighing more than 3500 kg is considered a heavy vehicle (MFOM 2003). Typically, for two-lane, two-direction single carriageway roads, it is assumed that each lane supports half of the total heavy vehicles. Additionally, the Spanish pavement design guide proposes pavement sections according to these eight traffic categories (MFOM 2003). Therefore, this research grouped road segments and developed models according to these traffic categories because road segments in the same category withstand similar levels of traffic loads, and, additionally, they support them with structurally similar pavement structures (flexible or semi-rigid pavements).

Table 1. Traffic categories in Spain (MFOM 2003)

Traffic category	**Heavy vehicles/day***	**Traffic category**	**Heavy vehicles/day***
T00	≥ 4000	T31	200 > hv/day ≥ 100
T0	4000 > hv/day ≥ 2000	T32	100 > hv/day ≥ 50
T1	2000 > hv/day ≥ 800	T41	50 > hv/day ≥ 25
T2	800 > hv/day ≥ 200	T42	25 > hv/day

Note: *Heavy vehicles/day in the lane with the highest number of heavy vehicles

With regard to the other affecting factor on pavement deterioration, environmental agents, it was decided to analyse pavement segments in the same climate region. Roads in the same climatic region are similarly affected by the meteorology. In Spain, there are five main climate areas: a hot-summer Mediterranean, warm Mediterranean, oceanic, semi-arid, and warm-summer continental climate. The climate areas indicated in the pavement design guide (MFOM 2003) were selected. The pavement structure and properties of the materials are designed according to the rainfall areas (Figure 1) and according to the summer thermal areas (Figure 2). The area of the region of Castile and Leon, and surrounding provinces were selected as an example of the same climatic zone according to these two criteria: the pluviometry area number five combined with the median summer thermal area. The selected area corresponds, approximately, to the provinces of Leon, Burgos, Soria, Segovia, and Ávila in the region of Castile and Leon, and the provinces of Madrid (region of Madrid) and Guadalajara (region of Castile-La Mancha). The resulting area of analysis is shown in Figure 3, where the rainfall area number five and the median summer thermal area are superimposed for the selected zone of analysis.

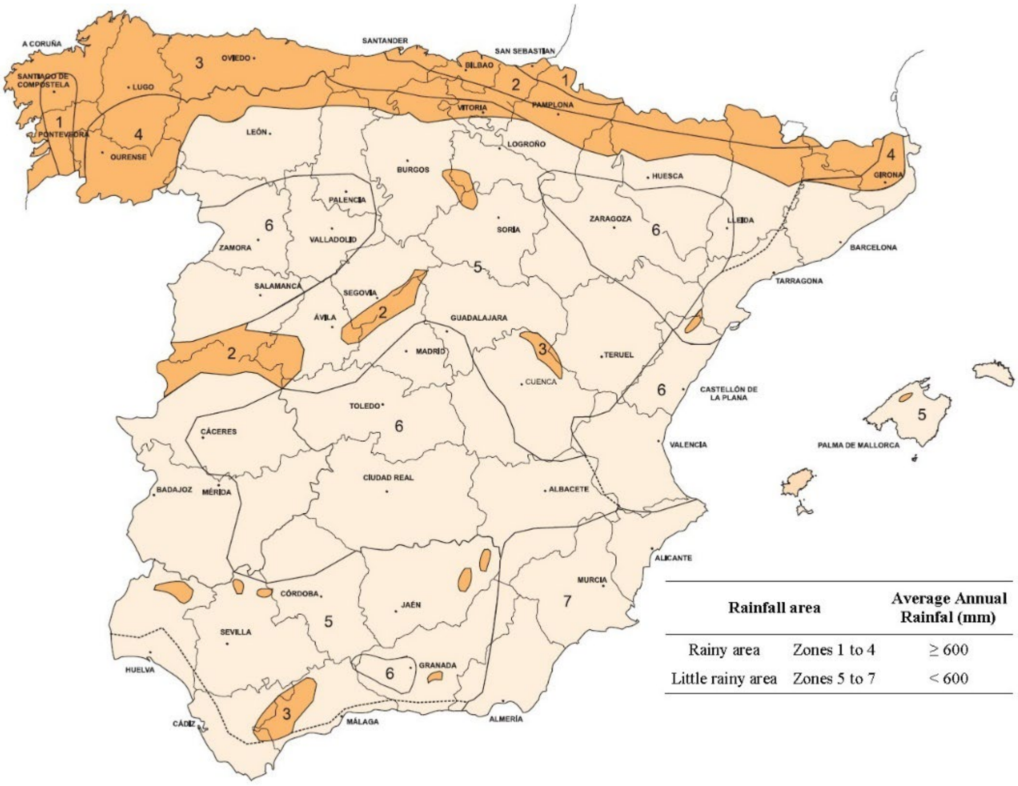

Figure 1. Rainfall areas in Spain according to MFOM (2003).

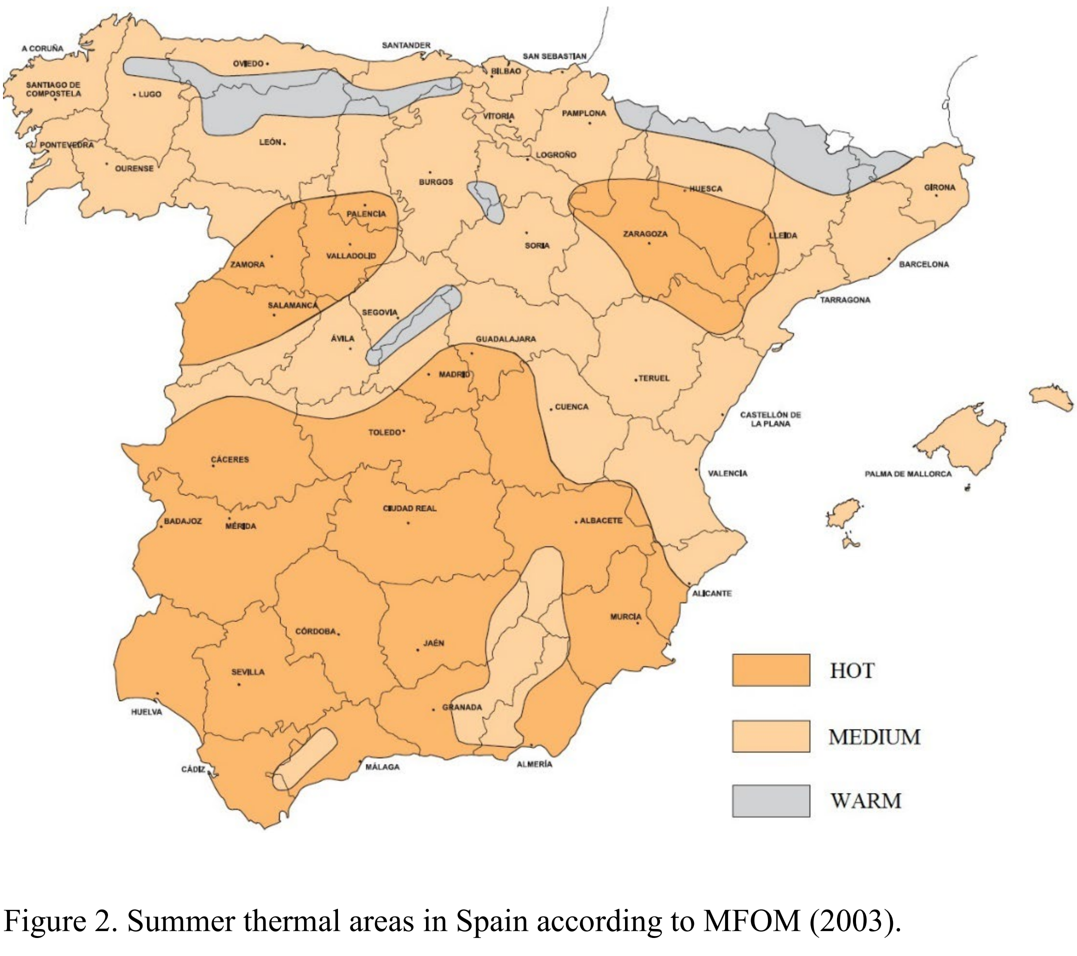

Figure 2. Summer thermal areas in Spain according to MFOM (2003).

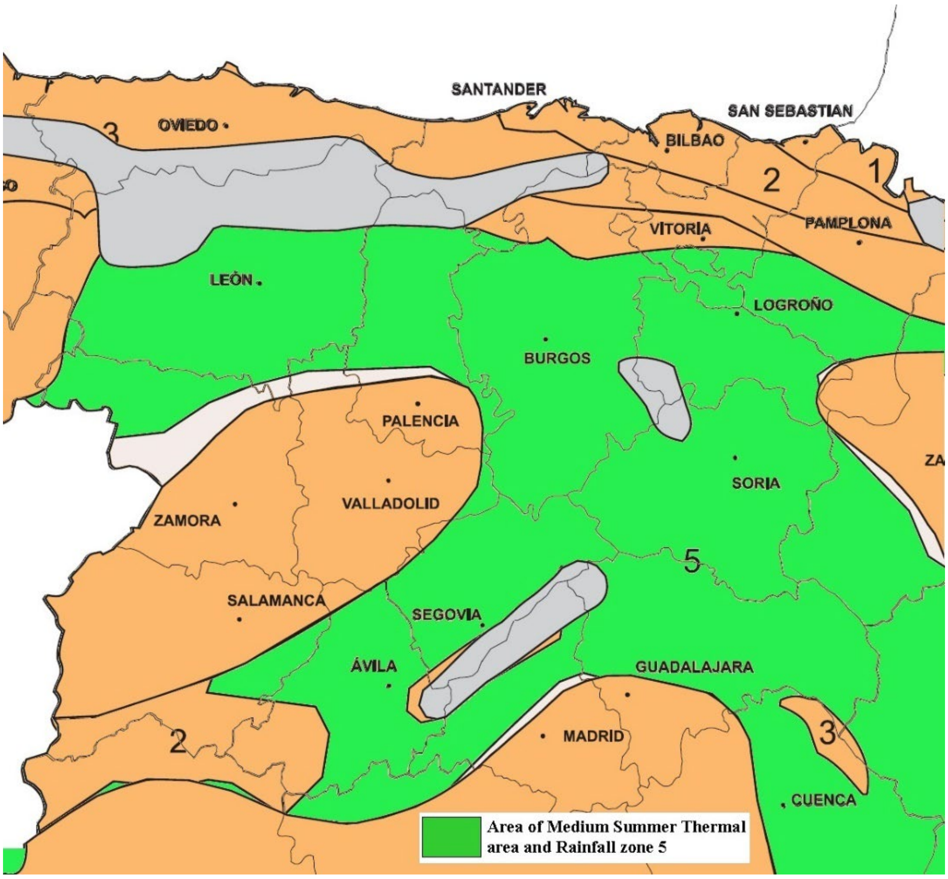

Figure 3. Superposition of rainfall area number 5 and the median summer thermal area.

Unlike deterministic models, traffic volumes and climatic factors cannot be introduced in probabilistic models. Therefore, to introduce these affecting factors it was necessary to limit the analysed roads to a homogeneous climatic area and traffic category (road segments were classified according to their traffic category, based on the AADTHV. This assumes that the road segments within the same traffic category and exposed to the same meteorology deteriorate similarly. The traffic categories for the analysed roads, by province, are presented in Table 2.

Each transition from one data collection period to another is analysed and, after observing the total road segments (of 100 m) remaining in the same condition state or transitioning to another state are counted by means of Equation (10), and then, the

Transition Probability Matrix is obtained. For developing the TPMs, supplementary hypotheses were assumed, which are described in next subsection.

Table 2. List of analysed roads by province, and traffic categories in each road

Province	Road Code (Stretch)	Traffic categories	Length (km)	Number of segments
Burgos	N-120 (Burgos – Logroño)	T1	55.7	1116
	N-232 (Pancorbo – N-629)	T2, T31, T32	40.3	672
	N-234 (Burgos – Soria)	T2, T31	75.5	1522
	N-622 (Quintana del Puente – Lerma)	T31, T41	32.0	640
	N-623 (Ubierna – N-232)	T2, T31, T32, T41	73.0	1462
	N-627 (Burgos – Aguilar de Campoó)	T2	49.9	998
Leon	N-120 (León – Astorga)	T1, T2	40.8	820
	N-621 (León – Devesa)	T41	16.8	338
	N-625 (Mansilla de las Mulas – Cistierna)	T2, T31, T32	43.8	882
	N-630 (León – La Robla)	T2	21.7	436
Segovia	N-110 (Segovia – San Esteban de Gormaz)	T2, T31, T32	96.6	1846
	N-110 (Segovia – Villacastín)	T1, T2, T31, T32	40.6	904
	N-601 (Adanero – Valladolid)	T1, T2	27.2	556
Soria	N-110 (Segovia – San Esteban de Gormaz)	T31, T32	23.4	470
	N-111 (Soria – Logroño)	T2, T31, T32	35.8	678
	N-111 (Soria – Medinaceli)	T1, T2	74.7	1532
	N-122 (Soria – Aranda de Duero)	T1, T2, T31	87.5	1784
	N-122 (Soria – Tarazona)	T1, T2	52.3	1046
	N-234 (Burgos – Soria)	T2, T31, T32	57.3	1146
	N-234 (Soria – Catalayud)	T2, T31, T32, T42	47.1	950
Guadalajara	N-320 Venturada – A2 (Guadalajara)	T1, T2, T31	25.3	524
Madrid	N-320 Venturada – A2 (Guadalajara)	T31, T32	24.1	482
TOTAL			**1041.4**	**20804**

Assumed supplementary hypotheses

Additional hypotheses were developed to create the TPMs for the region.

(1) Pavement type. The developed model is conducted with roads with bituminous layers as the top layer, which is the typical top layer material in Spain. Pavements can be semi-rigid or flexible, depending on whether the base materials are treated with cement, lime, fly ash, etc. or not, respectively. Although it is known that pavement can have different performance according to their typology (Xuan *et al*. 2012, Ismail *et al*. 2014, Linares-Unamunzaga *et al*. 2019, Fedrigo *et al*. 2021, Pérez-Acebo *et al*. 2021), the pavement design guide provides equivalent solutions for each traffic category, which can withstand the same level of traffic loads, regardless the typology (MFOM 2003).

(2) Condition states. The number and range of the condition states is debated thoroughly by Yamany *et al*. (2021). In the literature, examples from 3 to 20 condition states can be found, depending on the index/parameter applied, available data, and the required level of detail. Some authors recommend that condition states must have equal range widths for all the condition states, as proposed by Butt *et al*. (1987), Odoki and Kerali (2000), and Pérez-Acebo *et al*. (2018). Additionally, each condition state must have enough observations for achieving statistically significant results (Perez-Acebo *et al*. 2019). Adedimila *et al*. (2009) and Pérez-Acebo *et al*. (2018) proposed a range of 2 m/km, starting from 0 m/km (0-2; 2-4; 4-6; 6-8; 8-10). However, for Spanish national roads, high values are rare and values over 4 m/km are scarce, representing slightly over 5% of observations. Unlike Yamany *et al*. (2021) who gave a large percentage to the worst condition state (over 20% of all the possibilities, ranging from 4 to 10 condition states), it was preferred to assign a minimum of 5% to the worst CS because once a section arrives to that CS it cannot be in a worse CS. Finally, a constant width of 1 m/km was adopted, starting from 0 and ending at

4, as shown in Table 3. Selected values are also in accordance with the usual gradation of pavement segments according to IRI. While IRI values below 1.0 m/km are considered as excellent values, which are observed when a newly paved segment is opened to traffic, values may reach up to 2.0 m/km in new roads. Values over 3.0 m/km alert highway administrations in Spain that M&R should be conducted soon, and the limit of 4.0 m/km, is usually regarded as the maximum level. As a result, very few sections, approximately 5%, had values over 4 m/km.

Table 3. Established condition states, based on IRI values

Condition State	Definition	IRI value (m/km)	Average IRI value of the range
1	Excellent	IRI ≤ 1	0.5
2	Very good	1 < IRI ≤ 2	1.5
3	Good	2 < IRI ≤ 3	2.5
4	Poor	3 < IRI ≤ 4	3.5
5	Failed	IRI > 4	4.5

(3) Pavement age. As discussed in subsection 3.1, the age of the pavement segments included in this study is unknown (including when the segment was opened to traffic or when a major rehabilitation was carried out). In fact, almost all of the segments were in service before 1990 because they were national roads that were paved with asphalt during the decades of 1950s – 1980s, forming the backbone of the Spanish road network. Then, in the 1980s freeways were constructed parallel to these old routes, reducing the traffic volumes in the two-lane two-direction single carriageways. Hence, the analysed segments are old roads that are rehabilitated when they reach inadequate IRI values cyclically. Therefore, segments with variable ages are analysed, giving a broad spectrum of

ages, ranging from recently rehabilitated sections (1 year old) to old sections (approximately 20 years old). Thus, as variable ages are combined for calculating the TPMs, the disadvantage of the homogeneous Markov models is resolved. In staged-homogeneous Markov models, TPMs are developed according to different pavement ages (Butt *et al*. 1987, Yamany *et al*. 2021). In fact, for example, for pavements with ages between 1 and 6-7 years, very few sections are in the worst conditions (IRI > 3 m/km, in our case), so the transitions for calculating those elements of the matrix are scarce. Since pavements sections of different ages are included, older sections will show how they evolve with IRI values over 3 m/km. This combination of ages will result in a clear evolution of pavements when arriving to certain condition values, regardless their age.

(4) Improvements: maintenance and rehabilitation (M&R) activities. As detailed previously, sections are cyclically rehabilitated, and those works were also conducted during the analysis period, from 2004 to 2011. In section 2, this paper documented that improvements in pavement condition cannot occur unless they are associated with maintenance and rehabilitation (M&R) activities. Therefore, if a section showed an improvement in the IRI value compared with the previous year, that transition in condition state was not considered. A limit of 0.4 m/km was established for the identification of M&R activities. If a section's IRI value was reduced by more than 0.4 m/km after one year, the completion of a rehabilitation work was assumed and that transition was not included in the deterioration modelling. However, after that improvement, the deterioration observed in the subsequent step time was used for the TPM calculation. In fact, that evolution valuably showed the short-term performance after a rehabilitation

work. A sample of the data can be seen in Table 4. Case 1 shows deterioration from 2005 to 2006 and, hence, that deterioration is considered. In 2007 the IRI improves more than 0.4 m/km, implying that rehabilitation efforts were carried out. Hence, that transition was not included in the modelling. Then, the evolution from 2007 to 2009 can be considered again, as indicated in Table 4, and from then, onwards. Pérez-Acebo *et al*. (2018) applied a value of 4.0 m/km for considering a rehabilitation work in the section in Moldova, where roads are not rehabilitated until they have high IRI values, typically above 5.0 or 6.0 /km, and then, a high improvement is achieved. Conversely, in the case of Spain, pavements are renewed when the IRI is approximately 3.0 m/km, which is considered the first threshold (before arriving at the unacceptable Spanish standard value of 4.0 m/km). Normally, rehabilitation works are not conducted in short stretches, of 200 to 400 m long, but in longer distances of more than 2 or 3 km. With those longer distances, although most of the 100 m-long segments might be damaged (with IRI values over 3.0 m/km), some of the pavement segment might have good enough values, for example, approximately 1.5 m/km, and in those cases, the improvement will not be as substantial. This example illustrates the reason to apply a relatively low value as a threshold for considering improvements due to rehabilitation.

Additionally, in the cases when a difference of two years appears between data collection, an improvement of 0.15 m/km is used to imply rehabilitation works. After two years, worse IRI values are expected. As seen in case 2 of Table 4, the improvement from 2007 to 2009 eliminates the transition, but the following one, from 2009 to 2011 was included in the modelling. The limit after two years was not established as 0.0 m/km because seasonal variations and measuring errors can appear, as commented in the next point.

Table 4. Examples of IRI values and their consideration in the proposed probabilistic analysis

Case	2005		2006			2007			2009			2011		
	IRI	Cons CS[a]	IRI	Cons CS[a]	Trans: (Y/N)[b]	IRI	Cons CS[a]	Trans: (Y/N)[b]	IRI	Cons CS[a]	Trans: (Y/N)[b]	IRI	Cons CS[a]	Trans: (Y/N)[b]
1	2.85	3	3.05	4	Y	1.85	2	N	1.97	2	Y	2.14	3	Y
2	2.20	3	2.55	3	Y	2.85	3	Y	2.10	2	N	2.41	2	Y
3	2.10	3	2.55	3	Y	2.19	3	Y	2.65	3	Y	3.07	4	Y
4	2.13	3	1.95	3	Y	2.21	3	Y	2.58	3	Y	2.89	3	Y

[a] Considered condition state
[b] Transition considered? (Yes/No)

(5) Improvements: measurement errors and seasonal variations. Although inertial profilometers as designed to measure in the wheel path of the lane, human or device errors can occur and data may not always be taken in the same place as in the previous measurement. Hence, improvements in IRI values in the same segment may be recorded, even if M&R works were not conducted since the previous measurement. This can happen in some 100 m-long segments, but if the entire length of the road is analysed, deterioration is typically observed (higher median IRI values in the second measurement). This may occur with probabilistic models, where all the segments are taken into consideration. Furthermore, improvements can be also reported due to seasonal variations. Although most of the measures were taken in summer, some of them were recorded in spring or in autumn which may result in some differences. Seasonal changes in asphalt concrete pavement profile occur mainly due to changes in volume of the subsurface layer. Perera and Kohn (2002) observed variations in

IRI values from 0.26 to 0.50 depending on the climate region using data from the LTPP (Long Term Pavement Performance) database. Consequently, with the aim of including improvements on IRI values due to possible human errors and seasonal variations, a limit of 0.40 m/km is considered. Hence, if an IRI value of a 100 m-long segment has an improvement lower than 0.40 (i.e., a difference between values of -0.40), it is not considered as an M&R action, but an improvement resulting from seasonal effects or measurement errors. Case 3 of Table 4 shows an example of this type. Between 2006 and 2007, an improvement of 0.36 is observed, but it is not registered as an M&R action and that transition is considered along with the following transitions. Sometimes, that improvement up to 0.40 may lead to a better condition state. In those cases, as shown in case 4 of Table 4, the previous CS is maintained (CS in 2006), even if the IRI value corresponds to a better one. In the subsequent year (2007), the IRI value is higher than in the first year (2005), so the assumed IRI deterioration continues as expected. This example clearly reflects a deviation due to a measurement error or seasonal variations.

(6) Traffic categories. TPMs are developed based on traffic categories in Spain (Table 1) and not for each road individually. Various traffic categories can appear in the same road, as shown in Table 2. Furthermore, the traffic category may change from one measurement year to the next year. In those cases for this study, the traffic category of the first year is considered as the reference for that transition; regardless if the traffic volume increases or decreases in subsequent time periods. Table 5 shows various examples. Even if the category is higher (case 1) or lower (case 2) during the ensuing year, the first year is what determines the category. As data collection were mainly conducted in the

summer, the traffic category of the first year is assumed to have circulated through the segment during the whole year (implying that that volume of vehicles really deteriorated the road), and the traffic volume of the following year only affected the pavement for the first half of the year. This assumption can address the recurrent disadvantage of the homogeneous Markov chains. The same TPM is considered for the entire analysed period, regardless of the traffic volume variations. Following this assumption, if traffic volumes increase, which can be expected with population or economic growth, this will be reflected with a classification in a higher traffic category and, hence, another TPM must be used for forecasting the pavement evolution.

Table 5. Examples of cases with different traffic categories from one year to another.

Case	2006		2007		Considered traffic category
	Heavy veh./day	Traffic category	Heavy veh./day	Traffic category	
1	85	T32	110	T31	T32
2	250	T2	180	T31	T2

Results and discussion

Following the procedure explained in subsection 2.2 and assuming the hypotheses presented in section 3, IRI values from the roads in Table 2 were introduced in the modelling. Firstly, each traffic category of each road was individually analysed. Although the TPMs created for each step time (one year- or two year-duty cycle) could be very different, if the TPMs for a duty cycle of one year were calculated, similar values were obtained. An example is shown in Figure 4. It is the traffic category T31 of the road N-234 in the province of Soria.

Number of segments transitioning from one condition state to another

From 2004 to 2006

	CS	1	2	3	4	5
52	1	48	4	0	0	0
82	2	0	66	16	0	0
48	3	0	0	41	7	0
12	4	0	0	0	9	3
5	5	0	0	0	0	5
199						

2006 to 2007

	CS	1	2	3	4	5
160	1	153	7	0	0	0
422	2	0	384	38	0	0
153	3	0	0	138	15	0
26	4	0	0	0	19	7
9	5	0	0	0	0	9
770						

2007 to 2009

	CS	1	2	3	4	5
106	1	90	16	0	0	0
213	2	0	210	3	0	0
12	3	0	0	12	0	0
0	4	0	0	0	0	0
0	5	0	0	0	0	0
331						

2009 to 2011

	CS	1	2	3	4	5
388	1	322	66	0	0	0
488	2	0	476	12	0	0
32	3	0	0	25	5	2
2	4	0	0	0	2	0
2	5	0	0	0	0	2
912						

Resulting Transition Probatility Matrix directly from the data

	CS	1	2	3	4	5
52	1	0.92	0.08	0.00	0.00	0.00
82	2	0	0.80	0.20	0.00	0.00
48	3	0	0	0.85	0.15	0.00
12	4	0	0	0	0.75	0.25
5	5	0	0	0	0	1.00
199						

	CS	1	2	3	4	5
160	1	0.96	0.04	0.00	0.00	0.00
422	2	0	0.91	0.09	0.00	0.00
153	3	0	0	0.90	0.10	0.00
26	4	0	0	0	0.73	0.27
9	5	0	0	0	0	1.00
770						

	CS	1	2	3	4	5
106	1	0.85	0.15	0.00	0.00	0.00
213	2	0	0.99	0.01	0.00	0.00
12	3	0	0	1.00	0.00	0.00
0	4	0	0	0	0.00	0.00
0	5	0	0	0	0	1.00
331						

	CS	1	2	3	4	5
388	1	0.83	0.17	0.00	0.00	0.00
488	2	0	0.98	0.02	0.00	0.00
32	3	0	0	0.78	0.16	0.06
2	4	0	0	0	1.00	0.00
2	5	0	0	0	0	1.00
912						

Equations (32), (34), (36), (38), and (39) | DIRECTLY | Equations (32), (34), (36), (38), and (39) | Equations (32), (34), (36), (38), and (39)

Resulting Transition Probability Matrix for a duty cycle of one year

	CS	1	2	3	4	5
52	1	0.96	0.04	0.00	0.00	0.00
82	2	0	0.90	0.11	0.00	0.00
48	3	0	0	0.92	0.08	-0.01
12	4	0	0	0	0.87	0.13
5	5	0	0	0	0	1.00
199						

	CS	1	2	3	4	5
160	1	0.96	0.04	0.00	0.00	0.00
422	2	0	0.91	0.09	0.00	0.00
153	3	0	0	0.90	0.10	0.00
26	4	0	0	0	0.73	0.27
9	5	0	0	0	0	1.00
770						

	CS	1	2	3	4	5
106	1	0.92	0.08	0.00	0.00	0.00
213	2	0	0.99	0.01	0.00	0.00
12	3	0	0	1.00	0.00	0.00
0	4	0	0	0	0.00	0.00
0	5	0	0	0	0	1.00
331						

	CS	1	2	3	4	5
106	1	0.91	0.09	0.00	0.00	0.00
213	2	0	0.99	0.01	0.00	0.00
12	3	0	0	0.88	0.08	0.07
0	4	0	0	0	1.00	0.00
0	5	0	0	0	0	0.00
331						

Transitions considered to be added for the road in the same category

	CS	1	2	3	4	5
52	1	50	2	0	0	0
82	2	0	74	9	0	0
48	3	0	0	44	4	0
12	4	0	0	0	10	2
5	5	0	0	0	0	5
199						

+

	CS	1	2	3	4	5
160	1	153	7	0	0	0
422	2	0	384	38	0	0
153	3	0	0	138	15	0
26	4	0	0	0	19	7
9	5	0	0	0	0	9
770						

+

	CS	1	2	3	4	5
106	1	98	8	0	0	0
213	2	0	211	2	0	0
12	3	0	0	12	0	0
0	4	0	0	0	0	0
0	5	0	0	0	0	0
331						

+

	CS	1	2	3	4	5
388	1	353	35	0	0	0
488	2	0	482	6	0	0
33	3	0	0	28	3	2
2	4	0	0	0	2	0
0	5	0	0	0	0	0
911						

=

TOTAL CASES

	CS	1	2	3	4	5
706	1	654	52	0	0	0
1205	2	0	1151	55	-1	0
246	3	0	0	223	22	2
40	4	0	0	0	31	9
14	5	0	0	0	0	14
2211						

CS	1	2	3	4	5
1	0.93	0.07	0.00	0.00	0.00
2	0	0.96	0.05	0.00	0.00
3	0	0	0.90	0.09	0.01
4	0	0	0	0.78	0.22
5	0	0	0	0	1.00

Figure 4. Example of calculation of the global matrix of a road for a traffic category with variable duty cycles.

There were data from the years 2004, 2006, 2007, 2009, and 2011. The first row of matrices in Figure 4 shows the number of 100 m-long segments transitioning in each duty cycle (from 2004 to 2006, from 2006 to 2007, from 2007 to 2009, and from 2009 to 2011). The numbers one to five refer to the condition states (each matrix shows how many segments transition from the original condition state defined by the rows to the new condition state defined by the columns). The column outside the matrix to the left indicates the number of segments in each condition state in the first year and the matrix shows how those segments evolve in the subsequent time periods. From those data, TPMs can be calculated by dividing the element by the total of that row (shown in the number on the same row outside the matrix), as indicated in Equation (10), and those matrices are displayed in the second row of Figure 4. However, only the matrix from 2006 to 2007 remains identical in the third row because it represents the transition for a one-year duty cycle. The TPMs for a step time of one year for the other transitions (from 2004 to 2006, from 2007 to 2009, and from 2009 to 2011) must be calculated using Equations (32), (34), (36), (38), and (39), resulting in the matrices exhibited in the third row. In this row, all the TPMs indicate the transition for only one year. At this point in the process, the intermediate results show minor differences between the values at the same position in the matrix. For example, element p_{11} ranges from 0.96 to 0.91, or element p_{22} ranges from 0.90 to 0.99 and p_{33} from 0.9 to 1.0 but in the last values only 12 segments were available. It is logical that not all the matrices are identical. There are different quantities of segments in each transition (199, 770, 331, and 912) and the stochastic nature of pavements implies these differences are expected (Li *et al*. 1997, Hong 2014, Abaza 2016a). These results were promising and validated the assumed hypotheses for developing the TPMs.

Next, those matrices must be averaged, but not directly. As each transition contains a variable number of segments, the quantity of segments must be considered. Hence, the number of transitions must be recalculated using the developed TPMs for a one-year duty cycle (third row in Figure 4). In the case of the transition from 2006 to 2007, the resulting values (in the fourth row) are the same as in the first row. However, when calculating the cases in the fourth row for the transitions with a step time of two years (the first, third, and fourth column), different numbers of cases are obtained. In all the cases, the total number of segments in each CS at the first year of the transition is maintained, as shown in the columns next to the matrices. Finally, once the quantities of segments in each position of the matrix are calculated, they are added, resulting in the first matrix of the fifth row, which is just the addition of the matrices of the fourth row. With those values, the final matrix for road N-234 in the province of Soria for traffic category T31 is calculated, shown in green in the fifth row of Figure 4.

This process was repeated for each road and each traffic category. Low levels of variation were observed, except for situations with a low sample size (i.e., under 50 cases). In general, differences between values lower than 0.10 in the individual matrix elements within the same road and traffic category were observed.

Finally, once all the total cases and final TPMs for each road and TC were prepared, the total cases for each TC were added to create the final TPM for each traffic category. They are displayed in the first column of Figure 5 which can be considered as the average TPM of all the roads. To test the similarity between the results, the minimum and maximum values for each element p_{ij} in each road were obtained, with the aim of observing the maximum range between results of the analysed roads. As expected and previously explained, when the number of cases in one condition state is limited, a great difference can be observed. Therefore, cases from roads where the total number of cases in a row (in a condition state) is lower than 50, that road was not considered for calculating the range between the minimum and the maximum value of the element. However, those cases, even with sample sizes lower than 50, are computed in the calculation of the global matrix of the traffic category.

TRAFFIC CATEGORY	TOTAL NUMBER	CS	GLOBAL MATRIX 1	2	3	4	5	MINIMUM VALUES 1	2	3	4	5	MAXIMUM VALUES 1	2	3	4	5
T1	890	1	**0.852**	**0.139**	**0.009**	**0.000**	**0.000**	0.372	0.031	-0.002	-0.044	-0.001	0.937	0.611	0.039	0.001	0.044
	4300	2	0	**0.954**	**0.045**	**0.000**	**0.001**	0	0.931	0.013	-0.003	0.000	0	0.990	0.064	0.004	0.001
	797	3	0	0	**0.924**	**0.071**	**0.004**	0	0	0.854	0.028	0.000	0	0	0.972	0.122	0.024
	75	4	0	0	0	**0.814**	**0.186**	0	0	0	0.816	0.071	0	0	0	0.929	0.184
	6	5	0	0	0	0	**1.000**	0	0	0	0	1.000	0	0	0	0	1.000
TOTAL	6,068																
T2	3660	1	**0.834**	**0.166**	**0.000**	**0.000**	**0.000**	0.593	0.085	-0.004	0.000	0.000	0.915	0.407	0.005	0.007	0.004
	14569	2	0	**0.942**	**0.058**	**0.000**	**0.000**	0	0.793	0.019	-0.006	0.000	0	0.982	0.206	0.004	0.002
	3652	3	0	0	**0.922**	**0.076**	**0.003**	0	0	0.859	0.030	-0.005	0	0	0.971	0.146	0.012
	523	4	0	0	0	**0.847**	**0.153**	0	0	0	0.799	0.183	0	0	0	0.817	0.201
	133	5	0	0	0	0	**1.000**	0	0	0	0	1.000	0	0	0	0	1.000
TOTAL	22,538																
T31	1678	1	**0.848**	**0.152**	**0.000**	**0.000**	**0.000**	0.712	0.074	-0.003	0.000	0.000	0.926	0.290	0.004	0.000	0.000
	9524	2	0	**0.928**	**0.072**	**0.000**	**0.000**	0	0.778	0.023	-0.003	0.000	0	0.977	0.224	0.002	0.001
	2928	3	0	0	**0.917**	**0.080**	**0.003**	0	0	0.816	0.009	-0.008	0	0	0.986	0.173	0.011
	444	4	0	0	0	**0.879**	**0.121**	0	0	0	0.843	0.082	0	0	0	0.918	0.112
	80	5	0	0	0	0	**1.000**	0	0	0	0	1.000	0	0	0	0	1.000
TOTAL	14,654																
T32	363	1	**0.870**	**0.130**	**0.000**	**0.000**	**0.000**	0.816	0.089	0.000	0.000	0.000	0.911	0.185	0.000	0.000	0.000
	3648	2	0	**0.947**	**0.052**	**0.001**	**0.000**	0	0.857	0.000	-0.004	0.000	0	1.000	0.146	0.003	0.000
	1165	3	0	0	**0.940**	**0.059**	**0.001**	0	0	0.919	0.007	-0.001	0	0	0.993	0.079	0.001
	186	4	0	0	0	**0.899**	**0.101**	0	0	0	0.892	0.085	0	0	0	0.915	0.108
	29	5	0	0	0	0	**1.000**	0	0	0	0	1.000	0	0	0	0	1.000
TOTAL	5,390																
T41	26	1	**0.885**	**0.115**	**0.000**	**0.000**	**0.000**	0.880	0.120	0.000	0.000	0.000	0.880	0.120	0.000	0.000	0.000
	688	2	0	**0.920**	**0.078**	**0.002**	**0.000**	0	0.833	0.048	0.000	0.000	0	0.952	0.162	0.005	0.000
	494	3	0	0	**0.963**	**0.036**	**0.001**	0	0	0.950	0.034	-0.001	0	0	0.967	0.042	0.008
	66	4	0	0	0	**0.939**	**0.061**	0	0	0	0.965	0.035	0	0	0	0.965	0.035
	1	5	0	0	0	0	**1.000**	0	0	0	0	1.000	0	0	0	0	1.000
TOTAL	1,275																
T42	138	1	**0.940**	**0.060**	**0.000**	**0.000**	**0.000**										
	171	2	0	**0.988**	**0.012**	**0.000**	**0.000**										
	16	3	0	0	**0.935**	**0.065**	**0.000**										
	5	4	0	0	0	**0.895**	**0.105**										
	2	5	0	0	0	0	**1.000**										
TOTAL	332																

Figure 5. Global matrices for each traffic category, and minimal and maximum values for each element in the matrix for individual roads.

As observed in Figure 5, for traffic category T1, elements of rows 2, 3, and 4 do not show a great difference between the minimum and maximum values (less than 0.12, which means less than a 12% difference). This fact implies that similar deterioration patterns were observed for all the roads. In the case of p_{11}, the minimum value is quite low when compared with the maximum and the one in the global matrix. That value comes from a specific road (N-120 Burgos – Logroño); where few values were present. There were 99 segments in CS 1 and only 37 remained in CS 1. However, those 99 data points only represent around 11% of the data. For this road, the majority of the segments were close to the point of shifting its CS. This probabilistic nature is inherent to pavement performance. Moreover, values in CS 4 and CS 5 were obtained from relatively few points also (75 and 6, respectively). This indicates that for important roads (with a high TC), IRI values over 3.0 are not allowed, or are quickly rehabilitated, by the Ministry.

Similarly, for traffic category T2, all the elements except for the ones in the first row exhibited a range between the minimum and maximum values under 0.19, which can be considered a normal range between different roads. Elements in row 1, especially p_{11} and p_{12}, showed a difference over 0.30. Once again, those results come from a specific road, N-110 Segovia – San Esteban de Gornaz; where 129 data points were recorded in CS 1 and 77 remained in that condition state. Those 129 values only represent 3.5% of the total (3660). The TPM for traffic category T2 can be considered the most precise as it includes more than 22,500 transitions.

In traffic category T31, the main differences in the range are registered in the elements of the main diagonal and in p_{12} and p_{23}, with values of approximately 0.2. The range is not excessive, and the minimum values come from roads with fewer values in those condition states. In general terms, the global matrix is a reasonable balance between the maximum and minimum values.

Regarding traffic category T32, the difference between the minimum and maximum values is lower than 0.1, except for elements in row 2, because of a particular road (N-110 Segovia – San Esteban de Gornaz) which had all the road segments in CS 2 (287) remain in that CS with low values in the range of that CS (1.0 – 2.0 m/km). Apart from this element, the rest of the values are homogeneous.

For traffic category T41, the main differences in the range are in row 2 (p_{22} and p_{23}), lower than 0.12. In fact, the values for row 1 comes from a unique road, the only one with more than 50 data points in that CS. Homogeneity is observed for this category. Finally, the TPM for T42 is only obtained from a unique road, implying that the range of the values cannot be reliably evaluated. Very few data points are available because roads in the SRN have higher traffic volumes.

Sometimes, negative values can be observed in some matrix elements. They are the result of applying Equations (32), (34), (36), (38) and (39) to transitions of two years. However, these negative values are low, and they are compensated with positive values of other matrices.

In general, low proportions of segments in CS 4 and CS 5 can be observed, which means that the MTMAU tries to maintain them in an adequate state, implying that, if high IRI values are detected, over 3.0 m/km, M&R activities are conducted soon after.

Additionally, with the aim of validating the analysis, the developed matrices were applied to a new road (Figure 6). A perfect condition state vector was considered for the year 0, when it is opened to traffic, with 60% of section in CS 1 and 40% in CS 2, which can be a typical reference according to the established limits for IRI values in new roads (MFOM 2014, Mucka 2017). Hence, the resulting initial vector at $t = 0$ is presented in Equation (40)

$$A = \{0.60, 0.40, 0.00, 0.00, 0.00\} \tag{40}$$

The average values were calculated by multiplying the percentage of segments in each CS by the average value for that CS, which is the midpoint of the range. As shown in Table 3, with 4.5 as the value for CS 5, the segments in that condition are scarce in Spain. After 20 years, there is not a great difference between the average IRI values of the road, ranging from 2.355 for T32 to 2.704 for T31. This implies that adequate pavement structures are designed in the pavement design guide of Spain (MFOM 2003) because similar values are obtained after 20 years. For each annual average daily traffic of heavy vehicles (AADTHV), a reasonable solution is provided; not over dimensioning the layers nor by subdividing them.

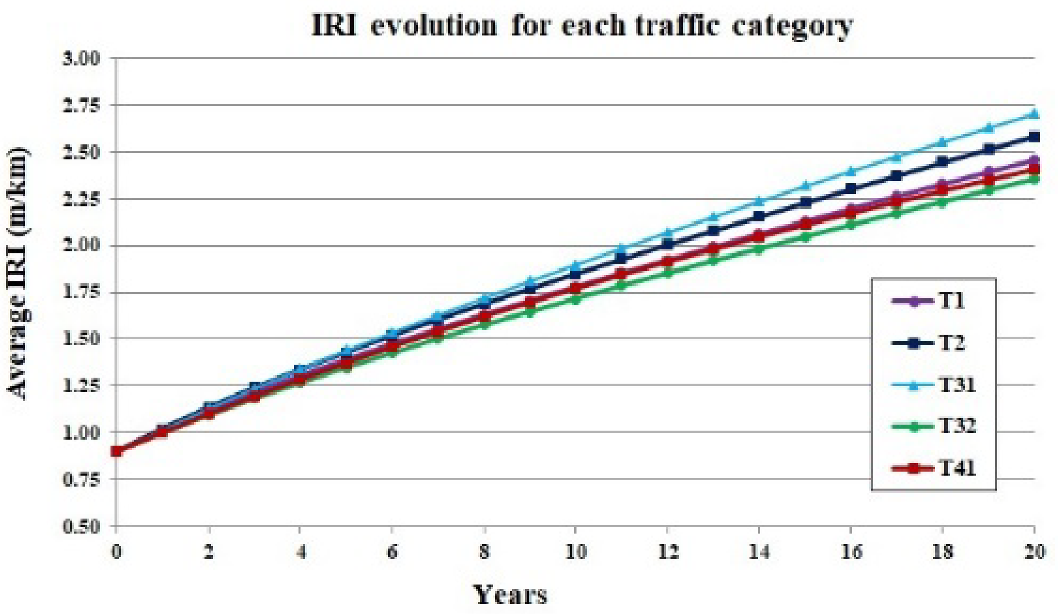

Figure 6. Evolution of an initial condition state for 20 years in each traffic condition.

A thorough analysis of when a certain percentage of segments arrive to a CS was also conducted. Figure 7 shows the percentages of sections in CS 1, CS 3, and CS 4 during a 20-year analysis period for each traffic category. Purple circles indicate when 20% of sections arrive to CS 3 (IRI values over 2.0 m/km), which can be an initial threshold, and when 5% of sections arrive to CS 4 (IRI values over 3.0 m/km), which can be regarded as a final threshold, because pavements over this value require an immediate intervention. Exact values are displayed in Table 6.

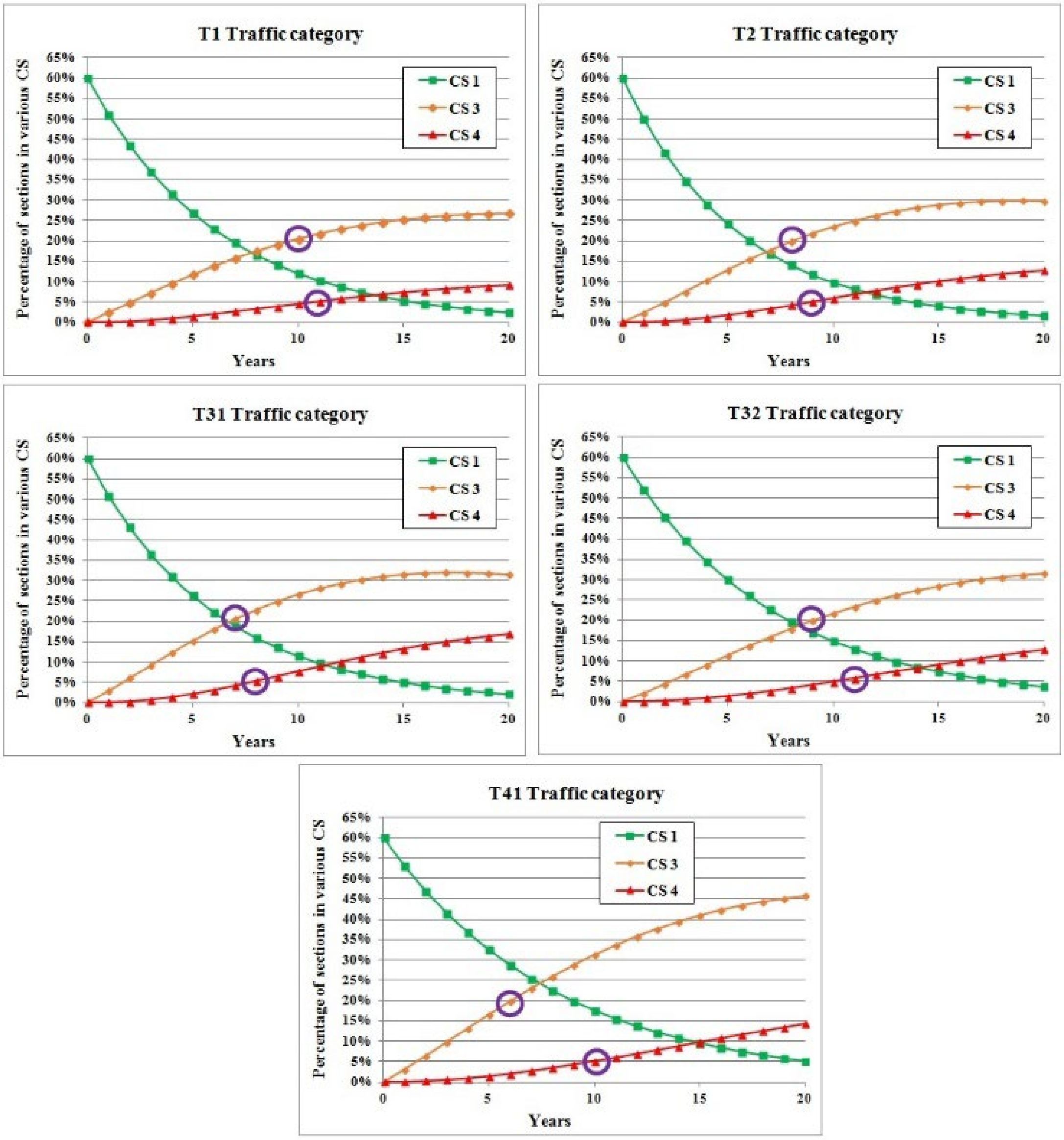

Figure 7. Percentages of sections in CS 1, CS 3 and CS 4 during a 20-year analysis period for each traffic category

Table 6. Years after a new segment is opened for achieving 20% of segments in CS 3 and 5% of segments in CS 4.

Traffic category	**20% of sections in CS 3 (2.0 < IRI ≤ 3.0 m/km)**	**5% of sections in CS 4 (3.0 < IRI ≤ 4.0 m/km)**
T1	10	11
T2	8	9
T31	7	8
T32	9	11
T41	7	10

As seen in Table 6, roads in T1 delay the arrival of unacceptable higher percentages in worse condition states, CS 3 and CS 4, more than the rest of the traffic categories; which is satisfactory, because greater mobility is supported on those roads. In general, the values are relatively similar, without great differences, underlining the previous idea: adequate pavement structures are proposed in the Spanish pavement design standard (MFOM 2003).

In summary, the TPMs for each individual road in each traffic category did not show a great difference, approximately 0.1 – 0.15, with maximum differences in atypical cases explained by characteristics of specific roads. However, this variability could be expected and is inherent to pavement performance, which is probabilistic in nature (Li *et al*. 1997, Tjan and Pitaloka 2005, Abaza 2016b). On the other hand, although homogeneous Markov chains were developed, the problem of increasing traffic volumes in the latter years of road life is solved establishing various traffic categories. If the traffic volume of a segment increases (or decreases) substantially, a different TPM must be adopted. Moreover, the necessity of developing staged-homogeneous Markov chains is overcome because for developing the matrices, pavements at different ages are included, averaging the results according to various pavement ages.

Conclusions

This paper demonstrated that it is feasible to develop pavement performance models by means of homogeneous Markov chains when pavement data collection is conducted with variable frequency. More specifically, IRI data collected every year or every two years were employed. This is a valuable insight because in practical scenarios, whether caused by workforce, equipment, or resource shortages, if a planned data collection schedule is disrupted, the data can still be used regardless of its inconsistent step time.

Homogeneous Markov chains mean that all the transitions are similar and the elements for a one-year step time can be calculated from a matrix obtained from data collected with a difference of two years. Improvements in the pavement condition cannot be observed unless a maintenance and rehabilitation activity is carried out, which allows for direct calculation of the matrix for a one-year step time, as shown in Equations (32), (34), (36), (38), and (39). Additionally, the model can be developed for any quantity of condition states.

Apart from the methodology to combine one and two-year duty cycles, it is necessary to assume two main hypotheses and other minor hypotheses. Firstly, segments must be from the same climatic area, which means that pavements are affected similarly by meteorological factors. Secondly, road segments must be classified according to their traffic volumes and various categories can be established.

Among the minor hypotheses, a threshold must be established for considering whether a rehabilitation work has been conducted. For a one-year duty cycle, an improvement of 0.40 m/km was established, and for a two-year step time, 0.15, as it is more probable to observe deterioration on roughness after two years. Improvements lower than those values were considered as seasonal variations or measurement errors. Additionally, if the traffic category during a transition changes; the traffic category of the first year is considered for that transition, which helps solve a recurrent problem in homogeneous Markov series. Generally, homogeneous TPMs are criticized because they cannot get the variable deterioration during the entire analysis period because traffic volumes may change. However, with this assumption of an initial categorization of traffic volumes, if traffic volumes increase, the segment will be included in another category for subsequent years. Moreover, as pavement segments with variable age are used for developing the model they include pavements of all ages, avoiding the necessity of using staged-homogeneous Markov chains.

Resulting TPMs with the same duty cycle time (one year) were approximately equal for similar roads. When the segments within the same traffic category from all the roads were compared, the range differences from the minimum to maximum values for each p_{ij} element were within normal values, approximately 0.1 and 0.15, with some exceptions resulting from characteristics of specific roads. Globally observed, those differences showed the stochastic nature of pavement performance, which is consistent with the findings of many authors. Final TPMs for each traffic category exhibited a satisfactory model for pavement data that are typically available as part of a PMS.

Finally, as relatively similar deterioration was observed for all the traffic categories, it was concluded that the pavements are adequately designed for each traffic category.

Declaration of interest: The authors report there are no competing interests to declare.

Data availability:

The data that support the findings of this study are openly available in the webpage of the Ministerio de Transportes, Movilidad y Agenda Urbana of the Spanish Government at https://www.mitma.gob.es/carreteras/datos-de-auscultacion-de-firmes-en-la-rce

References

AASHTO (1993). *Guide for design of pavement structures*. American Association of State Highway and Transportation Officials, Washington, DC, USA.

AASHTO (2012). *Pavement Management Guide, 2nd ed*. American Association of State Highway and Transportation Officials, Washington, DC, USA.

Abaza, K. A. (2004). Deterministic performance prediction model for rehabilitation and management of flexible pavement. *International Journal of Pavement Engineering*, 5 (2), 111-121.

Abaza, K. A. (2016a). Back-calculation of transition probabilities for Markovian-based pavement performance prediction models. *International Journal of Pavement Engineering*, 17 (3), 253-264.

Abaza, K. A. (2016b). Simplified staged-homogenous Markov model for flexible pavement performance prediction. *Road Materials and Pavement Design*, 17 (2), 365-381.

Abaza, K. A. (2017). Empirical approach for estimating the pavement transition probabilities used in non-homogenous Markov chains. *International Journal of Pavement Engineering*, 18 (2), 128-137.

Abaza, K. A., Murad, M. M. (2010). Pavement rehabilitation project ranking approach using probabilistic long-term performance indicators. *Transportation Research Record*, 2153 (1), 3-12.

Abdelaziz, N., Abd El-Hakim, R. T., El-Badawy, S. M., Afify, H. A. (2020). International Roughness Index prediction model for flexible pavements. *International Journal of Pavement Engineering*, 21 (1), 88-99.

Adedimila, A. S., Olutaiwo, A. O., Kehinde, O. (2009). Markovian probabilistic pavement performance prediction models for a developing country. *Journal of* Engineering and Applied Sciences, 4 (1), 13–26.

Alaswadko, N. H. A. (2016). *Deterioration modelling of granular pavements for rural arterial roads*. Ph. D thesis, Swinburne University of Technology, Melbourne, Australia.

Alaswadko, N., Hwayyis, K. (2022). An approach to investigate the supplementary inconsistency between time series data for predicting road pavement performance models. *International Journal of Pavement Engineering*, 1-15.

Alaswadko, N. H., Hassan, R. A., Mohammed, B. N. (2018). Empirical Roughness Progression Models of Heavy Duty Rural Pavements. *International Journal of Civil and Environmental Engineering*, 12(3), 301-307.

Alaswadko, N., Hassan, R., Meyer, D., Mohammed, B. (2019). Modelling roughness progression of sealed granular pavements: a new approach. *International Journal of Pavement Engineering*, 20(2), 222-232.

Albuquerque, F. S., Núñez, W. P. (2011). Development of roughness prediction models for low-volume road networks in northeast Brazil. *Transportation Research Record*, *2205*(1), 198-205.

Amin, M. S. R., Amador-Jiménez, L. E. (2016). Pavement management with dynamic traffic and artificial neural network: a case study of Montreal. *Canadian Journal of Civil Engineering*, *43* (3), 241-251.

ASTM (2020). *ASTM E867-06(2020). Standard terminology relating to vehicle-pavement systems*. ASTM International, West Conshohocken, PA, USA.

Bandara, N., Gunaratne, M. (2001). Current and future pavement maintenance prioriztization based on rapid visual condition evaluation. *Journal of Transportation Engineering*, 127 (2), 116-123.

BOE (Boletin Oficial del Estado) (2015). *Ley 7/2015, de 29 de septiembre, de Carreteras*. (BOE de 30 de septiembre de 2015). Boletín Oficial del Estado, Madrid, Spain,

Budras, J. (2001). *A synopsis on the current equipment used for measuring pavement smoothness*. Federal Highway Administration, Washington, DC, USA.

Butt, A. A., Shahin, M. Y., Feighan, K. J., Carpenter, S. H. (1987). Pavement performance prediction model using the Markov process, *Transportation Research Record*, 1123, 12-19.

Carnahan, J. V. (1988). Analytical framework for optimizing pavement maintenance. *Journal of Transportation Engineering*, 114 (3), 307-322.

Chatti, K. Zaabar, I. (2012). *Estimating the Effects of Pavement Condition on Vehicle Operating Costs*. NCHRP Report 720. Transportation Research Board, Washington, DC.

Chou, E., Pulugurta, H., Datta, D. (2008). *Pavement forecasting models*. Rep. No. FHWA/OH-2008/3. Columbus, OH: Ohio DOT.

Fani, A., Golroo, A., Ali Mirhassani, S., Gandomi, A. H. (2022). Pavement maintenance and rehabilitation planning optimisation under budget and pavement deterioration uncertainty. *International Journal of Pavement Engineering*, 23(2), 414-424.

Flintsch, G.W., McGhee, K.K. (2009). *Quality management of pavement condition data collection*. Washingto, DC: Transportation Research Board, National Cooperative Highway Research Program Report 401.

García de Soto, B., Bumbacher, A., Deublein, M, Adey, B. T. (2018). Predicting road traffic accidents using artificial neural network models. *Infrastructure Asset Management*, 5 (4), 132-144.

Gharieb, M., Nishikawa, T., Nakamura, S., Thepvongsa, K. (2022). Modeling of pavement roughness utilizing artificial neural network approach for Laos national road network. *Journal of Civil Engineering and Management*, 28 (4), 261-277.

Golroo, A., Tighe, S. (2009). Use of soft computing applications to model pervious concrete pavement condition in cold climates. Journal of Transportation Engineering, 135 (11), 791-800.

Gurney (1997). An introduction to neural networks. CRC Press, London.

Haas, R., Hudson, W.R., Zaniewski, J.P. (1994). Modern pavement management. Krieger, Melbourne, FL. USA.

Hassan, R., Lin, O., Thananjeyan, A. (2017a). A comparison between three approaches for modelling deterioration of five pavement surfaces. *International Journal of Pavement Engineering*, 18 (1), 26-35.

Hassan, R., Lin, O., Thananjeyan, A. (2017b). Probabilistic modelling of flexible pavement distresses for network management. *International Journal of Pavement Engineering,* 18 (3), 216-227.

Hillier, F.S. and Lieberman, G.J., 1990. *Introduction to operations research*. 5th ed. New York, NY: MacGraw-Hill.

Hong, F. (2014). Asphalt pavement overlay service life reliability assessment based on non-destructive technologies. *Structure and Infrastructure Engineering*, 10 (6), 767-776.

Hong, H. P., Wang, S. S. (2003). Stochastic modeling of pavement performance. *International Journal of Pavement Engineering*, 4 (4), 235-243.

Hudson, W. R., Haas, R., Pedigo, R. D. (1979). *Pavement management system development*. National Cooperative Highway Research Program Report 215, Highway Research Record 407. Transportation Research Board, National Research Council, Washington, DC, USA.

Hwayyis, K., Hassan, R., Fahey, M. T. (2022). Factors affecting performance of sprayed seals in rural Victoria. *International Journal of Pavement Engineering*, 23(7), 2278-2292.

Isaacson, D.L. and Madsen, R.W. (1976). *Markov chains: theory and applications*. New York, NY: Wiley.

Ismail, A., Baghini, M. S., Karim, M. R., Shokri, F., Al-Mansob, R. A., Firoozi, A. A., Firoozi, A. A. (2014). Laboratory investigation on the strength characteristics of cement-treated base. In Applied mechanics and materials (Vol. 507, pp. 353-360). Trans Tech Publications Ltd.

Jiang, Y., Yi, Y., Tian, T., Fan, J., Yuan, K., Deng, C., Xue, J. (2020). Development and application of skid resistance fog seal for pavements. *Coatings*, 10 (9), 867.

Justo-Silva, Ferreira, A., Flintsh, G. (2022). Review on machine learning techniques for developing pavement performance prediction models. *Sustainability*, 13 (9), 5248.

Kerali, H.R. and Snaith, M.S., 1992. *NETCOM: the TRL visual condition model for road networks*. Crowthorne, UK: Transport Research Laboratory, Contractor Report No. 921.

Kobayashi, K., Do, M., & Han, D. (2010). Estimation of Markovian transition probabilities for pavement deterioration forecasting. *KSCE Journal of Civil Engineering*, 14, 343-351.

Kulkarni, R. B. (1984). Dynamic decision model for a pavement management system. *Transportation Research Record*, 997, 11-18.

Lethanh, N., Adey, B. T. (2012). A hidden Markov model for modeling pavement deterioration under incomplete monitoring data. *International Journal of Civil and Environmental Engineering*, 6(1), 1-8.

Lethanh, N., Adey, B. T. (2013). Use of exponential hidden Markov models for modelling pavement deterioration. *International Journal of Pavement Engineering*, 14(7), 645-654.

Li, N., Haas, R., Xie, W.C., (1997). Development of a new asphalt pavement performance prediction model. *Canadian Journal of Civil Engineering*, 24 (4), 547–559.

Linares-Unamunzaga, A., Pérez-Acebo, H., Rojo, M., Gonzalo-Orden, H. (2019). Flexural strength prediction models for soil–cement from unconfined compressive strength at seven days. *Materials*, 12(3), 387.

Madanat, S., Mishalani, R., Ibrahim, W. H. W. (1995). Estimation of infrastructure transition probabilities from condition rating data. *Journal of Infrastructure Systems*, 1(2), 120-125.

MacLeod, D. R., Walsh, R. (1998, May). *Markov Modelling-A Case Study*. In Proc., 4th International Conference on Managing Pavements.Washington, DC: Transportation Research Board.

Mazari, M., Rodriguez, D. D. (2016). Prediction of pavement roughness using a hybrid gene expression programming-neural network technique. *Journal of Traffic and Transportation Engineering (English Edition)*, *3* (5), 448-455.

MFOM (Ministerio de Fomento) (2003). *FOM/3640/2003, de 28 de noviembre, por la que se aprueba la norma 6.1-IC "Secciones de Firme", de la Instrucción de Carreteras*. Ministerio de Fomento; Madrid, Spain.

MFOM (Ministerio de Fomento) (2014). *Orden FOM/2523/2014, de 12 de diciembre, por la que se actualizan determinados artículos del Pliego de Prescripciones Técnicas Generales para obras de carreteras y puentes, relativos a materiales básicos, a firmes y pavimentos, y a señalización, balizamiento y sistemas de contención de vehículos*. BOE de 3 de enero de 2015. Ministerio de Fomento; Madrid, Spain.

Mohammadi, A., Amador-Jimenez, L., Elsaid, F. (2019). Simplified pavement performance modeling with only two-time series observations: A case study of Montreal Island. *Journal of Transportation Engineering, Part B: Pavements*, 145 (4), 05019004.

MTMAU (Ministerio de Transportes, Movilidad y Agenda Urbana) (2022). *Catálogo Oficial de Carreteras. Longitud de carreteras de la RCE por tipo de vía. Situación a fecha: 31/12/2021*. Ministerio de Transportes, Movilidad y Agenda Urbana: Madrid, Spain. Accessed on 26/02/2023 https://www.mitma.gob.es/carreteras/catalogo-y-evolucion-de-la-red-de-carreteras

Mubaraki, M. (2010). *Predicting deterioration for the Saudi Arabia Urban road network*. Thesis (PhD) University of Nottingham.

Múčka (2017). International Roughness Index specifications around the world. *Road Materials and Pavement Design*, 18 (4), 929-965.

Odoki, J. B., Kerali, H. G. R. (2000). *Analytical framework and model descriptions. Highway Development and Management (HDM-4) Volume 4*. Washington, DC: World Road Association.

Olowosulu, A. T., Kaura, J. M., Murana, A. A., Adeke, P. T. (2022). Development of framework for performance prediction of flexible road pavement in Nigeria using Fuzzy logic theory. *International Journal of Pavement Engineering, 23* (11), 3809-3818.

Ortiz-García, J. J., Costello, S. B., Snaith, M. S. (2006). Derivation of transition probability matrices for pavement deterioration modeling. *Journal of Transportation Engineering*, 132 (2), 141-161.

Osorio-Lird, A., Chamorro, A., Videla, C., Tighe, S., Torres-Machi, C. (2018). Application of Markov chains and Monte Carlo simulations for developing pavement performance models for urban network management. *Structure and Infrastructure Engineering*, 14 (9), 1169-1181.

Perera, R.W. Kohn, S.D., 2002. *Issues in pavement smoothness: a summary report.* Washington, DC: Transportation Research Board, National Research Council, NCHRP Web Document 42. Contractor's Final Report.

Pérez-Acebo, H., Bejan, S., Gonzalo-Orden, H. (2018). Transition probability matrices for flexible pavement deterioration models with half-year cycle time. *International Journal of Civil Engineering*, 16 (9), 1045-1056.

Pérez-Acebo, H., Mindra, N., Railean, A., Rojí, E. (2019). Rigid pavement performance models by means of Markov Chains with half-year step time. *International Journal of Pavement Engineering*, 20 (7), 830-843.

Pérez-Acebo, H., Gonzalo-Orden, H., Findley, D. J., Rojí, E. (2020a). A skid resistance prediction model for an entire road network. *Construction and building materials*, *262*, 120041.

Pérez-Acebo, H., Linares-Unamunzaga, A., Rojí, E., Gonzalo-Orden, H. (2020b). IRI performance models for flexible pavements in two-lane roads until first maintenance and/or rehabilitation work. *Coatings*, *10* (2), 97.

Pérez-Acebo, H., Gonzalo-Orden, H., Findley, D. J., Rojí, E. (2021). Modeling the international roughness index performance on semi-rigid pavements in single carriageway roads. *Construction and building materials*, *272*, 121665.

Pérez-Acebo, H., Montes-Redondo, M., Appelt, A., & Findley, D. J. (2022a). A simplified skid resistance predicting model for a freeway network to be used in a pavement management system. *International Journal of Pavement Engineering*, https://doi.org/10.1080/10298436.2021.2020266

Perez-Acebo, H. Romo-Martín, A., Findley, D. (2022b). Spatial distribution and the facility evaluation of the service and rest areas in the toll motorway network of the European Union. *Applied Spatial Analysis and Policy*, 15 (3), 821-845.

Pulugurta, H., Shao, Q., Chou, Y. J. (2009). Pavement condition prediction using Markov process. *Journal of Statistics and Management Systems*, 12 (5), 853-871.

Salas, M. A., Pérez-Acebo, H., Calderón, V., Gonzalo-Orden, H. (2018). Bitumen modified with recycled polyurethane foam for employment in hot mix asphalt. *Ingeniería e Investigación*, 38 (1), 60-66.

Salas, M. A., Pérez-Acebo, H. (2018). Introduction of recycled polyurethane foam in mastic asphalt. *Gradevinar*, 70 (5), 403-412.

Sandra, A. K., Sarkar, A. K. (2013). Development of a model for estimating International Roughness Index from pavement distresses. *International Journal of Pavement Engineering*, *14*(8), 715-724.

Sayers, M.W., Gillespie, T.D., Queiroz, C.A.V. (1986a) *International Experiment to Establish Correlations and Standard Calibration Methods for Road Roughness Measurement; Technical Paper Number 45.* The World Bank: Washington, DC, USA.

Sayers, M.W., Gillespie, T.D., Paterson, W.D.O. (1986b). *Guidelines for conducting and calibrating road roughness measurements. Technical Paper Number 46*. The World Bank: Washington, DC, USA.

Sayers (1995). On the calculation of International Roughness Index from longitudinal road profile. *Transportation Research Record*, 1051, 1-12.

Shahnazari, H., Tutunchian, M. A., Mashayekhi, M., Amini, A. A. (2012). Application of soft computing for prediction of pavement condition index. *Journal of Transportation Engineering*, 138 (12), 1495-1506.

Sidess, A., Ravina, A., Oged, E. (2021). A model for predicting the deterioration of the pavement condition index. *International Journal of Pavement Engineering*, 22 (13), 1625-1636.

Smith, K., Ram, P. (2016). Measuring and specifying pavement smoothness. Federal Highway Administration (FHWA): Washington, DC, USA.

Smith, K. L., Smith, K. D., Evans, L. D., Hoerner, T. E., Darter, M. I., Woodstrom, J. H. (1997). *Smoothness Specifications for Pavements. NCHRP Web Document 1 (Project 1-31)*. Transportation Research Board, Washington, DC.

Sollazzo, G., Fwa, T. F., Bosurgi, G. (2017). An ANN model to correlate roughness and structural performance in asphalt pavements. *Construction and Building Materials*, *134*, 684-693.

Soncim, S. P., de Oliveira, I. C. S., Santos, F. B., Oliveira, C. A. D. S. (2018). Development of probabilistic models for predicting roughness in asphalt pavement. *Road Materials and Pavement Design*, 19 (6), 1448-1457.

Tabatabaee, N., Ziyadi, M. (2013). Bayesian approach to updating Markov-based models for predicting pavement performance. *Transportation Research Record*, 2366 (1), 34-42.

Thomas, O. Sobanjo, J., 2012. Comparison of Markov chain and semi-Markov models for crack deterioration on flexible pavements. *Journal of Infrastructure Systems*, 19 (2), 186–195.

Tjan, A., Pitaloka, D. (2005). *Future prediction of pavement condition using Markov probability transition matrix*. In Proceedings of the Eastern Asia Society for Transportation Studies (Vol. 5, pp. 772-782).

Uddin, W. (2006). *Pavement Management Systems*. In Fwa, T.F. (ed.). The Handbook of Highway Engineering, Taylor & Francis. Boca Raton, FL, USA.

Wang, K., Zaniewski, J., Way, G. (1994). Probabilistic behavior of pavement. *Journal of Transportation Engineering*, 120, 358-375.

Wen, T., Ding, S., Lang, H., Lu, J. J., Yuan, Y., Peng, Y., Chen, J., Wang, A. (2022). Automated pavement distress segmentation on asphalt surfaces using a deep learning network. *International Journal of Pavement Engineering,* https://doi.org/10.1080/10298436.2022.2027414

Xuan, D. X., Houben, L. J. M., Molenaar, A. A. A., Shui, Z. H. (2012). Mechanical properties of cement-treated aggregate material: A review. *Materials & Design*, 33, 496-502.

Yamany, M. S., Abraham, D. M., Labi, S. (2019). *Probabilistic modeling of pavement performance using Markov chains: A state-of-the-art review*. In ASCE International Conference on Transportation & Development, Virginia, USA.

Yamany, M. S., Saeed, T. U., Volovski, M., Ahmed, A. (2020). Characterizing the performance of interstate flexible pavements using artificial neural networks and random parameters regression. *Journal of Infrastructure Systems*, 26 (2), 04020010.

Yamany, M. S., Abraham, D. M. (2021). Hybrid approach to incorporate preventive maintenance effectiveness into probabilistic pavement performance models. *Journal of Transportation Engineering, Part B: Pavements*, 147 (1), 04020077.

Yamany, M. S., Abraham, D. M., Labi, S. (2021). Comparative analysis of Markovian methodologies for modeling infrastructure system performance. *Journal of Infrastructure Systems*, 27 (2), 04021003.

Yang, J., Gunaratne, M., Lu, J. J., Dietrich, B. (2005). Use of recurrent Markov chains for modeling the crack performance of flexible pavements. *Journal of Transportation Engineering*, 131 (11), 861-872.

Yang, J., Lu, J. J., Gunaratne, M., Dietrich, B. (2006). Modeling crack deterioration of flexible pavements: Comparison of recurrent Markov chains and artificial neural networks. *Transportation Research Record*, 1974 (1), 18-25.

Zang, K., Shen, J., Huang, H., Wan, M., Shi, J. (2018). Assessing and mapping of road surface roughness based on GPS and accelerometer sensors on bicycle-mounted smartphones. *Sensors*, 18 (3), 914.

Ziari, H., Sobhani, J., Ayoubinejad, J., Hartmann, T. (2016). Prediction of IRI in short and long terms for flexible pavements: ANN and GMDH methods. *International journal of pavement engineering*, 17 (9), 776-788

Table 1. Traffic categories in Spain (MFOM, 2003).

Traffic category	Heavy vehicles/day*	Traffic category	Heavy vehicles/day*
T00	≥ 4000	T31	200 > hv/day ≥ 100
T0	4000 > hv/day ≥ 2000	T32	100 > hv/day ≥ 50
T1	2000 > hv/day ≥ 800	T41	50 > hv/day ≥ 25
T2	800 > hv/day ≥ 200	T42	25 > hv/day

Note: *Heavy vehicles/day in the lane with the highest number of heavy vehicles

Table 2. List of analysed roads by province, and traffic categories in each road

Province	Road Code (Stretch)	Traffic categories
Burgos	N-120 (Burgos – Logroño)	T1
	N-232 (Pancorbo – N-629)	T2, T31, T32
	N-234 (Burgos – Soria)	T2, T31
	N-622 (Quintana del Puente – Lerma)	T31, T41
	N-623 (Ubierna – N-232)	T2, T31, T32, T41
	N-627 (Burgos – Aguilar de Campoó)	T2
Leon	N-120 (León – Astorga)	T1, T2
	N-621 (León – Devesa)	T41
	N-625 (Mansilla de las Mulas – Cistierna)	T2, T31, T32
	N-630 (León – La Robla)	T2
Segovia	N-110 (Segovia – San Esteban de Gormaz)	T2, T31, T32
	N-110 (Segovia – Villacastín)	T1, T2, T31, T32
	N-601 (Adanero – Valladolid)	T1, T2
Soria	N-110 (Segovia – San Esteban de Gormaz)	T31, T32
	N-111 (Soria – Logroño)	T2, T31, T32
	N-111 (Soria – Medinaceli)	T1, T2
	N-122 (Soria – Aranda de Duero)	T1, T2, T31
	N-122 (Soria – Tarazona)	T1, T2
	N-234 (Burgos – Soria)	T2, T31, T32
	N-234 (Soria – Catalayud)	T2, T31, T32, T42
Guadalajara	N-320 Venturada – A2 (Guadalajara)	T1, T2, T31
Madrid	N-320 Venturada – A2 (Guadalajara)	T31, T32

Table 3. Established condition states, based on IRI values

Condition State	Definition	IRI value (m/km)	Average IRI value of the range
1	Excellent	IRI ≤ 1	0.5
2	Very good	1 < IRI ≤ 2	1.5
3	Good	2 < IRI ≤ 3	2.5
4	Poor	3 < IRI ≤ 4	3.5
5	Failed	IRI > 4	4.5

Table 4. Examples of IRI values and their consideration in the proposed probabilistic analysis

Case	2005		2006			2007			2009			2011		
	IRI	**Cons CS[a]**	**IRI**	**Cons CS[a]**	**Trans: (Y/N)[b]**	**IRI**	**Cons CS[a]**	**Trans: (Y/N)[b]**	**IRI**	**Cons CS[a]**	**Trans: (Y/N)[b]**	**IRI**	**Cons CS[a]**	**Trans: (Y/N)[b]**
1	2.85	3	3.05	4	Y	1.85	2	N	1.97	2	Y	2.14	3	Y
2	2.20	3	2.55	3	Y	2.85	3	Y	2.10	2	N	2.41	2	Y
3	2.10	3	2.55	3	Y	2.19	3	Y	2.65	3	Y	3.07	4	Y
4	2.13	3	1.95	3	Y	2.21	3	Y	2.58	3	Y	2.89	3	Y

[a] Considered condition state

[b] Transition considered? (Yes/No)

Table 5. Examples of cases with different traffic categories from one year to another

Case	2006		2007		Considered traffic category
	Heavy veh./day	**Traffic category**	**Heavy veh./day**	**Traffic category**	
1	85	T32	110	T31	T32
2	250	T2	180	T31	T2

Table 6. Years after a new segment is opened for achieving 20% of segments in CS 3 and 5% of segments in CS 4.

Traffic category	**20% of sections in CS 3 (2.0 < IRI ≤ 3.0 m/km)**	**5% of sections in CS 4 (3.0 < IRI ≤ 4.0 m/km)**
T1	10	11
T2	8	9
T31	7	8
T32	9	11
T41	7	10

Figure 1. Rainfall areas in Spain according to MFOM (2003)

Figure 2. Summer thermal areas in Spain according to MFOM (2003)

Figure 3. Superposition of rainfall area number 5 and the median summer thermal area

Figure 4. Example of calculation of the global matrix of a road for a traffic category with variable duty cycles

Figure 5. Global matrices for each traffic category, and minimal and maximum values for each element in the matrix for individual roads

Figure 6. Evolution of an initial condition state for 20 years in each traffic condition

Figure 7. Percentages of sections in CS 1, CS 3 and CS 4 during a 20-year analysis period for each traffic category