IMPROVING BRT ROUTE DESIGN THROUGH CODE: THE CASE OF BOGOTÁ'S BRT SYSTEM, TRANSMILENIO

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ABSTRACT

This study presents the methodology used by TRANSMILENIO S.A. –the company in charge of managing the BRT lines in Bogotá, Colombia– to renew its operational design. TRANSMILENIO S.A. is increasing its bus fleet by acquiring higher capacity bi-articulated buses. This process demanded a redefinition of the frequencies, stops, and type of bus for all routes in the system. Therefore, a line scheme was selected based on the number of trips between corridors. A non-linear optimization model was then used to establish an initial route design that could satisfy passenger demand in the morning rush hour.

Five algorithms were implemented to adjust the initial route design to the actual restrictions, namely, fleet and bus capacity at stops. The algorithms modified the type of bus employed, the defined stops of routes, and eliminated low demand routes. Different alternatives were tested for the execution order of algorithms. The alternative to implement was selected based on compliance of the defined restrictions. Algorithms were iteratively run. Between iterations, a trip assignment model was used to adjust route frequencies based on demand.

The execution order of algorithms, its impact on the level of service indicators, and compliance of restrictions are part of this study's discussion. Particular relevance and effects of each algorithm are also analyzed. Both methodology and results might be tested, adapted, and improved in further studies by other BRT systems as a reference or input for their own route design process.

1. INTRODUCTION

Bus Rapid Transit systems have proven to be a sustainable solution for cities' mobility and economic development. This means of transport based on segregated lanes for bus circulation reduces the use of private cars, improves air quality, mitigates traffic accident rates, and generates employability (WRI, 2013). For its part, the city of Bogotá is responsible for 30% of the Colombian PIB and is one of the principal population center in the country (Hernández, 2020). TransMilenio is the name of Bogotá´s BRT system, one of the largest BRT systems in the world (WRI Brasil Ross, 2021). This system is managed by TRANSMILENIO S.A. a company owned by several public authorities of the city and is in charge of planning and managing the public transport system in Bogotá (which also includes regular bus routes and cable cars). Furthermore, this company does not own or operate buses but assigns routes to private transport operators.

Transport operators are privately-owned companies that own the buses and manage the hiring of drivers, and the maintenance, repair, washing, and operation of the buses. Transport operators are typically selected through a bidding process (including price per traveled kilometer and other factors) and awarded term-limited concessions to operate. City administration is currently making considerations to create a public-owned Transport operator.

Within the bidding process, a collection operator is also selected whose duty lies in the ticket purchase and the required hiring and maintenance for box-office operations. Tickets are sold in the form of electronic cards which are purchased by users in box-offices and “loaded” with a balance in currency.

The ticket value is then subtracted from the balance at the entrance of BRT stations or at non-BRT bus doors. The collection operator is also in charge of providing and maintaining the required electronic equipment in buses and stations, and the databases of user’s balance.

On the other hand, the city administration oversees the construction and maintenance of the infrastructure. Some infrastructure (e.g., parking lots, maintenance, washing, and facilities) is given to the operators to use and maintain during concessions. The city administration also provides capital to subsidize the system reducing the fare paid by users. Some financial institutions are also involved in saving resources and distributing them among the involved parties.

1.1 Operational Design

Bogotá city requires basic infrastructure and resilience services able to face the emerging challenge of growth. Hence, TRANSMILENIO S.A. performed two priority public bids (TMSA, 2018) which incremented the number of buses in the system.

The new fleet was divided into 67% bi-articulated and 33% articulated buses. The entity set off the design of a renovated operational design which apart from establishing the operable conditions for the new fleet, must guarantee general attention of Bogotá's BRT demanded trips. Likewise, BRT systems, unlike rail schemes, have a set of operational parameters that must be defined for service coverage. Particularly, BRT system has higher flexibility since its infrastructure can be adapted to the existing needs of a given space (CEPAL, 2012) and is not tied to rail infrastructure. Therefore, when planning the scheme of routes for this type of system, the followed variables must be considered:

- The number of routes offered per corridor or line.
- The defined path for each route.
- The stops within those routes.
- The frequencies with which of the routes to attend the trip demand.
- The type of bus to be used for each route.

These variables highlight the requirement to generate a process in which all are analyzed in a holistic and coordinated methodology. A methodology that leads to the execution of the process called Operational Design.

2. INITIAL SOLUTION

2.1 Existing methodologies

Planning public transport has been widely analyzed in the literature. In this section, two types of methodologies are presented: the first one related to a regular bus routes design, and general guidelines specifically for a BRT system.

In the first case, the principal process to evaluate a design is based on three main steps according to (Ceder, 2007): Initially, a feasible solution is proposed based on the different OD pairs with higher demand.This will result in a first approximation of the model.
Then, these groups of routes are evaluated based on the attended demand. Subsequently, an evaluation of frequencies and network performance is executed according to selected indicators such as fleet size, rules for insertion or elimination of nodes in existing routes, among others. Finally, it is obtained an improved solution after removing, dividing, or interchanging the routes.

The second one comprises a BRT Planning guide (ITDP, 2017) which shows the guidelines for executing the service planning. In the document, mainly in chapter 6, the different indicators and considerations for basic service planning are presented. Some of these are saturation, speed, travel time, service frequency and headways, direct and express services. However, these guidelines are presented from a general strategic perspective. In the present document, it is presented a combination of both methodologies.

2.2 Lines selection

One of the main objectives with the accomplishment of the new operational design was to migrate from one current system divided into trunk lines to one with lines, similar to urban rail transport systems like metros. Since Bogotá's BRT system currently has eleven trunk lines, analyzing all the possible lines becomes an unfeasible task because of communication to users, design complexity, and efficiency (not all the lines would have a significant number of travels).

To generate an initial approximation of this design, the first duty was to analyse the trip demand between all the origin-destination pairs in the possible lines to retain the most important ones, as mentioned by Ceder (2007). The trip demand used was the morning peak hour (6:30 to 7:30) estimated in a study for the integral structuring of the trunk component operation of TransMilenio System (Temporary Union SDG-PHR-KPMG, 2018).

Furthermore, in this contract, the lines with a demand of 6.000 trips per day and up to 9.000 trips were kept as "main lines", a step that was followed in this evaluation. Nevertheless, this process left some original trunk lines outside the proposed principal lines. To circumvent this issue, the lines with higher demand among the ones including the left-behind trunk lines were added to the "principal lines" set. At the end of the process, 22 lines were considered as main lines.

2.3 Optimization model and preliminary design

For each of the major lines selected a simple optimization model was developed in Excel to create a set of "seed routes" implemented within the following steps.

All the models constrained the maximum number of resulting routes to five (except for two lines for which the demand was too high and hence the number of routes was greater), all with a similar frequency (between nine and twenty bus/hour) while maximizing the number of biarticulated buses and avoiding leaving any station with no service. To define the objective function, all the origin-destination pairs in the line were ranked following the number of trips, and the pairs with more trips than the percentile 95, were marked. The objective function was defined as a maximization of the number of principal O-D pairs covered with no need for transfers.

3. ADITIONAL RESTRICTIONS

The preliminary design, which resulted from the initial optimization model required to be adjusted to the existing restrictions to be implemented successfully. These restrictions are described below.

3.1 Available buses

The resulting operational design should require no more buses than the available in the system. This parameter is given by the total amount of buses in the system minus a percentage of buses which is receiving maintenance or are kept in reserve to attend emergencies during operations. In TRANSMILENIO S.A. 2% of the total fleet is reserved for these purposes. This led to 736 articulated and 1263 bi-articulated buses available to operate.

3.2 Bus type restrictions at stations

After the acquisition of the new bi-articulated buses, this type of vehicle will constitute the majority in Bogotá’s BRT system. However, a bi-articulated bus commands larger stations, stop points, and doors. However, not every station in the system is adequate to operate with this vehicle type. The resulting design must ensure that there are no bi-articulated bus routes designed to stop in stations without the adequations required.

3.3 Bus station capacity

TransMilenio stations have one or more gates (stop points) for buses to stop in each direction. The number of buses that can stop at the same gate in a period is limited. When the number of buses increases above this limit, queues might form with a higher frequency, which can cause delays. Using traffic simulations and field observations TRANSMILENIO S.A. had previously defined capacity values for each station (buses per hour per direction). This was the greater restriction, and after consulting with the company team responsible for this data, a station was considered to comply if the number of buses was within 110% of the defined capacity value, otherwise, the station is considered to be saturated.

3.4 Headway restrictions

Headway between buses on the same route has a direct impact on user waiting time, and therefore, on the perceived level of service. To avoid long waiting times, TRANSMILENIO S.A. has established a seven (7) minute limit for the maximum headway allowed. This limit must be observed even when route demand could be served with a higher headway, therefore, complying with this condition might require more buses. A limit of two (2) minutes for the minimum headway allowed has also been established to avoid bus bunching.

4. DEVELOPED ALGORITHMS

To adjust the preliminary operational design to the current restrictions, four algorithms were developed that iteratively modified the initial scenario. A transit assignment model was used for the evaluation of the different supply scenarios. The model used was previously developed for TRANSMILENIO S.A. as part of the consultancy by Temporary Union SDG-PHR-KPMG (2018) and uses EMME transport planning software. Algorithms were implemented using python scripts. For more information on transit assignment models refer to Ortúzar and Willumsen (2011), chapter 10.6.

4.1 Relevant OD pairs inclusion

Although the initial optimization model sought to maximize the number of DO pairs that were directly connected, the solution obtained did not directly connect (that is, by the same route and without the need for transfers) all the pairs. In particular, some of the OD pairs with high trip numbers were not directly connected. Consequently, the first algorithm seeks for the OD pairs with the highest number of trips in the origin-destination matrix and connects them. OD pairs with several trips within a percentile that is received as a parameter are searched for.

Finally, once the largest OD pairs have been found, those that are not directly connected, a route is searched that passes through the corresponding origin and destination stations, and the mentioned route is used to connect the OD pair. In case of finding several routes, the route with the lowest volume of passengers at the station of origin is used. If no routes are found, the OD pair remains without direct connection. This algorithm is executed once (not iterative).

4.2 Capacity adjustment in stations

This algorithm was developed to reduce the formation of queues in the stations of the system and adjust to the measured capacity of the stations. The algorithm takes the current design, and searches for stations that are saturated (as defined in section 3.3) in any of the two directions, then eliminates the stop of the route with the least number of total movements (ascents and descents in both directions). Therefore, the selected route will no longer stop at this station.

After eliminating a route stop for each of the saturated stations, an assignment and headway adjustment (as described in 4.5) is carried out. The stop elimination process is repeated, continuing iteratively. The algorithm ends when the percentage of saturated stations falls below a defined threshold or is 0 (when there are no saturated stations), or when the maximum iteration defined number is reached.

4.3 Change to biarticulated type

Followed by the execution of any of the algorithms, it might result that the obtained operational design does not comply with the available fleet restriction. A particular case is that the design required more articulated buses than those available, and less bi-articulated than those available.

It is also possible that highly demanded routes that are candidates for using bi-articulated buses (for their higher passenger capacity) could not be initially designed with this vehicle type. This is caused now that the restriction of infrastructure. Hence, after executing the algorithm for capacity adjustment in stations, those routes were debugged to suspend stop in stations with biarticulated restrictions.

With those cases in mind, an algorithm was designed to seek those routes that are being served by articulated buses and stops only in stations equipped for bi-articulated buses. The algorithm modifies the type of bus of all the routes found from articulated to bi-articulated and performs a frequency adjustment and assignment process. This is not an iterative process, and all the routes found are modified at once.

4.4 Fleet adjustment

For the other cases in which the available fleet restriction is breached (even after executing the algorithm for changing to bi-articulated bus type), two algorithms were developed. These algorithms are designed to be executed consecutively.

4.5 Change to articulated type

The first algorithm seeks to comply with the restriction of bi-articulated buses and is executed when the current operational design requires more bi-articulated buses than those available. In each iteration, it looks for the route of bi-articulated buses with the lowest demand (fewer total boardings) and modifies it to articulated bus type. After each iteration, an assignment and headway adjustment (as described in 4.5) is carried out. The algorithm continues to iterate until the design meets the available fleet of bi-articulated buses.

4.6 Routes Elimination

After completing the first algorithm, and if the available articulated bus fleet restriction unfulfilled, the second algorithm will search for the articulated bus route with the least demand (least number of total boardings) and eliminate it. The algorithm continues to iterate until the design meets the available fleet of articulated buses. After each iteration, an assignment and headway adjustment (as described in 4.5) is carried out.

4.7 Assignment and headway adjustment

As stated above, between the iterations of the algorithms a process or assignment and headway adjustment was executed. This process was implemented to improve transit assignment results and adjust route frequencies to actual demand conditions. It consisted of three steps, as described below:

- The first step was to equal the headway from all routes. This was done in preparation for a transit assignment process, to equalize the perceived waiting time for the user. The subsequent transit assignment was therefore reliant on in-vehicle time and transfers, and less dependent on headway results from previous iterations.
- The second step consisted of a transit assignment using EMME’s optimal strategies (Spiess and Florian 1989) extended transit assignment algorithm, followed by a headway adjustment which was the third step. Headway adjustment consisted of calculating the frequency required for each route to be able to transport the passenger demand at the segment with the higher load; this was done by dividing hourly passenger demand at the highest loaded segment between bus passenger capacity.

> The resulting value was the required route frequency (in buses per hour) from which headway in minutes could be obtained. Capacity values used for articulated and biarticulated types were 160 and 260 passengers, respectively.

A relatively simple methodology for transit assignment was used, as the effects of vehicle capacity, parallel lines, and overlapping routes were not addressed. An alternative for the future improvement of this methodology is the use of more sophisticated assignment models, such as those developed by Verbas and Mahmassani (2016), or Schmöcker and Fonzone (2011).

4.8 Special route protection

In addition to all system routes, it exists a set of routes that stop in all stations along their path. These are known in the city as “easy” routes. There’s typically one of these routes for each main line in the system.

Due to their low speed, these routes often have a lower demand, and therefore relatively high headways. The existence of these routes usually ensures that all stations are interconnected, and helps new or occasional users to easily use the system.

Likewise, to prevent any of the algorithms to erase or modify these routes, a functionality was implemented to “protect routes” by marking them using a dummy variable in the software. Marked routes could be subjected to changes in the type of bus used, and their frequencies were adjusted to demand in every iteration (section 4.5) but were ignored by algorithms that eliminated routes or route stops.

Initially, only “easy” routes were protected. However, as planning process progressed, this functionality proved to be useful and was extended to other routes. The reason is briefly explained in section 7.

5. PERFORMANCE INDICATORS

To evaluate the performance of the developed algorithms, the overall design, and the resulting routes, were based on six indicators previously selected. Indicators were calculated between every iteration. These variables cover different parameters as bus fleet, waiting times in stations, and time in vehicles, among others. A description of each indicator will be shown below.

5.1 Bus fleet

After the renovation of the bus fleet, the quantity of articulated and bi-articulated buses available to operate is limited: 736 and 1263 respectively. These are target values as described in section 3.1.

5.2 In-vehicle time

In the transit assignment process (INRO, 2020), a time in vehicle is generated for each iteration. This is obtained by multiplying the transit times by a perception factor. These values were compared with the actual values corresponding to the base scenario.

5.3 Waiting time

Like in-vehicle time indicator, this indicator is obtained through the transit assignment algorithm by INRO (2020). This corresponds to the weighted average of passengers waiting times. These values were also compared with the actual values corresponding to the base scenario.

5.4 Average boardings

Indicates the average number of boardings passengers have to make to reach their destinations, including transfers. Values close to one indicate a low number of transfers in the system. Similar to the time in vehicle indicator, this variable is obtained by the transit assignment algorithm. These values were also compared with the actual values corresponding to the base scenario.

5.5 Saturated stations

This indicator shows the number of saturated stations (as defined in section 3.3) in relation to the total stations in the system. The indicator is presented as a percentage.

5.6 Generalized cost

This corresponds to the impedance function in the EMME algorithm. The total impedance is the sum of perceived time and cost components (total waiting times, total boarding times/costs, in-vehicle times/costs, auxiliary transit times/costs) where the perceived component is the actual component multiplied by the corresponding perception factor (INRO, 2020).

6. DEVELOPMENT PROCESS AND EXECUTION ORDER

Initially, only Fleet Adjustment (FA) algorithm (section 4.4) and Capacity Adjustment in Stations (CAS) algorithm (section 4.2) were developed. FA was intended to precede CAS, but this intended execution order proved inadequate. As a result, the execution order was inverted and further issues were identified, which lead to the development of the other two algorithms.

Scenarios 351 and 361 were both built using the preliminary design as the starting point (section 2.2). For scenario 351 FA was executed before CAS. When issues with scenario 351 were identified, scenario 361 was created with the execution order inverted. Issues with the first scenario are evident when compared to the second one, as is shown below.

For scenario 351, Change to Articulated Type (CAT) algorithm (section 4.4.1) was performed from iterations 1 to 8, when the bi-articulated fleet restriction was fulfilled. Route elimination (RE) algorithm (section 4.4.2) was subsequently performed until iteration 22, and CAS was then executed until iteration 42. At iteration 42 CAS maximum iteration number was reached without achieving convergence.

For scenario 361, CAS was executed from iterations 1 to 21, when the desired convergence was reached (5% of stations saturated). CAT was performed until iteration 30 when the bi-articulated fleet restriction was fulfilled. RE was then executed until iteration 50 when the maximum iteration number was reached (20 iterations).

Figure 1 shows the required number of bi-articulated buses along algorithm execution. This figure shows that CAT had the desired effect by reducing the number of required bi-articulated buses. However, required buses also vary when CAS is executed.

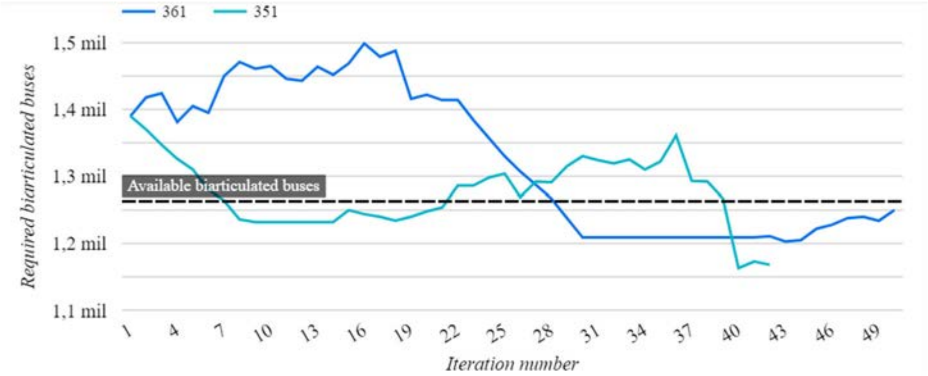


Figure 1: Required number of biarticulated buses throughout algorithm execution, scenarios 361 and 351

Figure 2 shows the required number of articulated buses along algorithm execution. This figure also shows the desired effect of RE algorithm in reducing the number of required articulated buses. It can be observed that required articulated buses also vary when CAS is executed.

Figure 2: Required number of articulated buses throughout algorithm execution, scenarios 361 and 351

By comparing both scenarios it was clear that progress obtained by FA algorithm could easily be undone by CAS. On the other hand, the inverse relation was not true. As shown in figure 3, the Percentage of saturated stations was effectively reduced by CAS and it was not significantly increased when executing FA. It was then decided to execute FA after CAS.

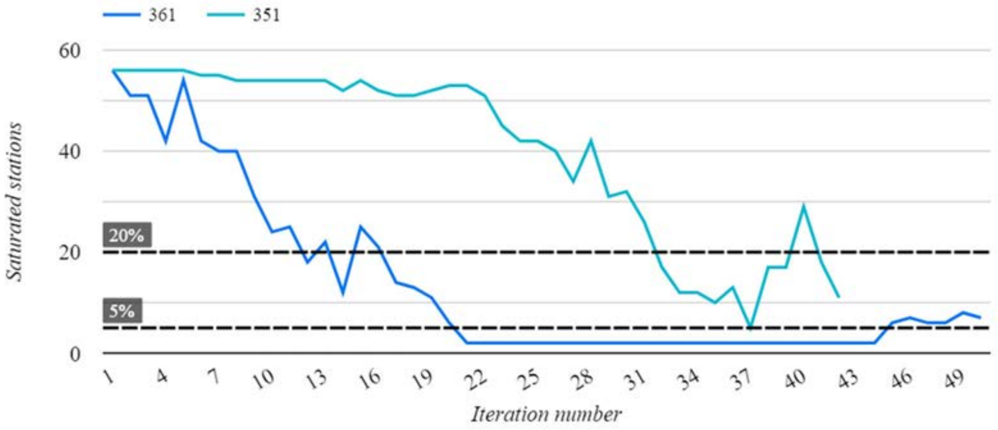
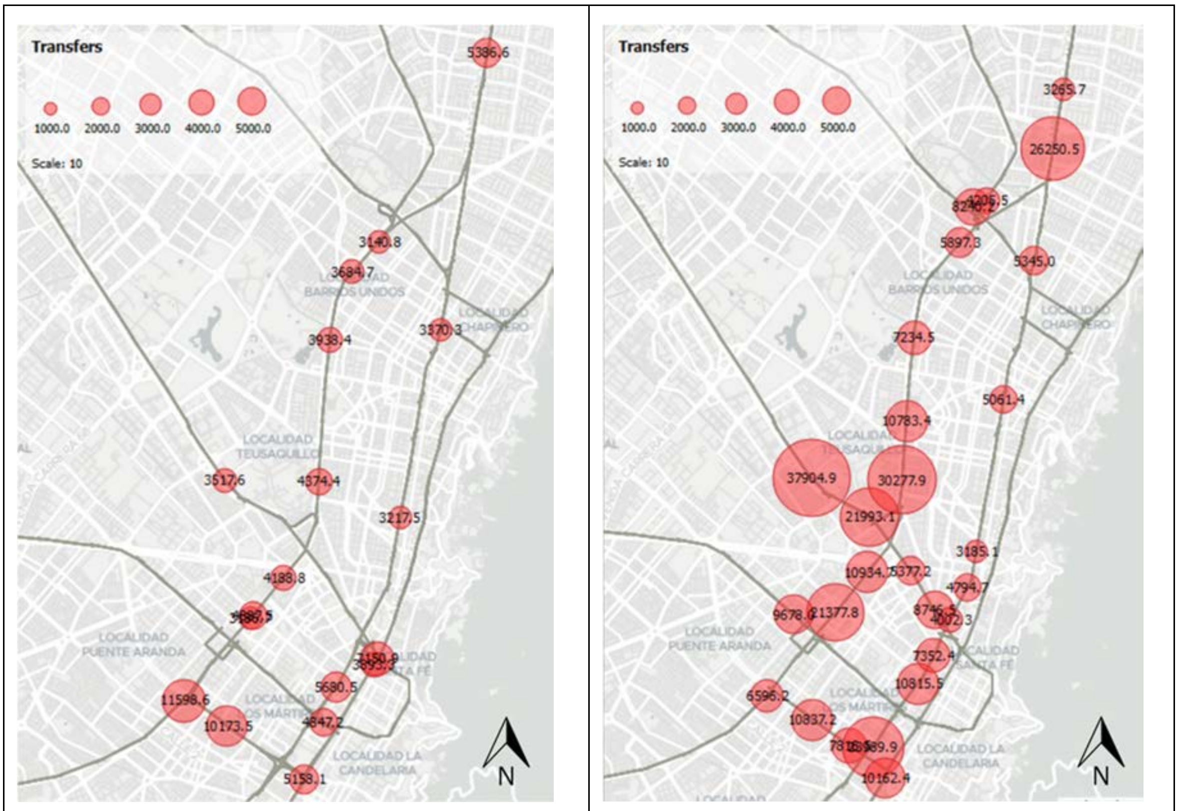


Figure 3: Percentage of saturated stations throughout algorithm execution, scenarios 361 and 351



a. b.

Figure 4: Stations with more than 1000 transfers base scen. (a) and scen. 361 (b)

FA and CAS algorithms were effective at complying with system restrictions. However, since both eliminated routes and routes stops, more transfers were required in the system. Even though for some stations the number of transfers diminished, for others, it augmented dramatically. "Calle 100" station, for example, went from nearly 2.350 transfers to 26.250; more than 11 times the initial value.

To address this problem, Relevant OD Pairs Inclusion (RPI), and Change to Biarticulated Type (CBT) algorithms (sections 4.1 and 4.3) were developed. CBT was executed between CAS and FA and two options were evaluated for the execution of RPI. RPI was run immediately after CAS in scenario 403, and before CAS (at the beginning of the whole process) in scenario 413.

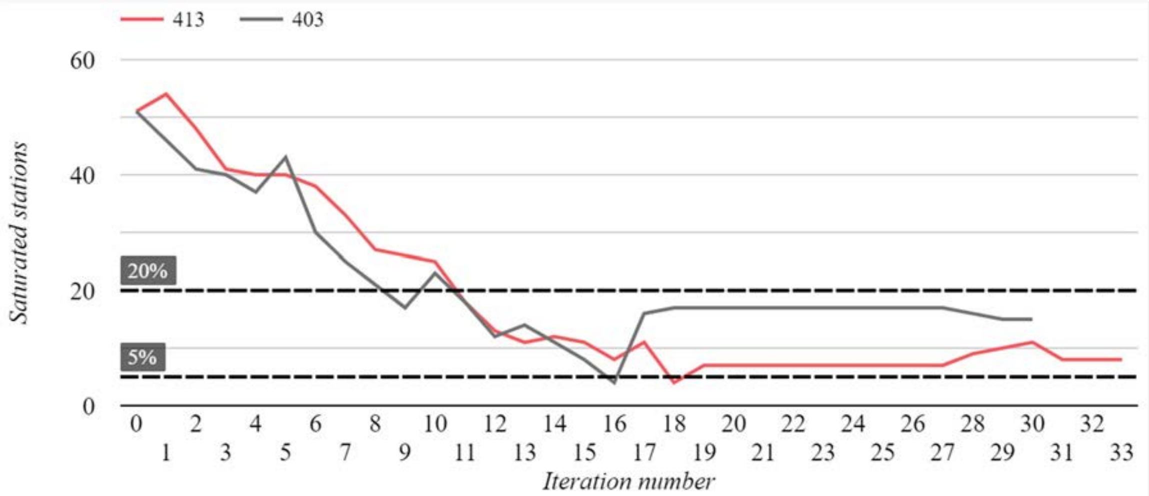


Figure 5: Percentage of saturated stations throughout algorithm execution, scenarios 413 and 403.

Except for the percentage of saturated stations, there were no significant differences in performance indicators between scenarios 403 and 413. Figure 5 shows that some of the progress achieved by CAS was undone by RPI in scenario 403 (Iteration 16 to 17). It was then decided to execute RPI before CAS.

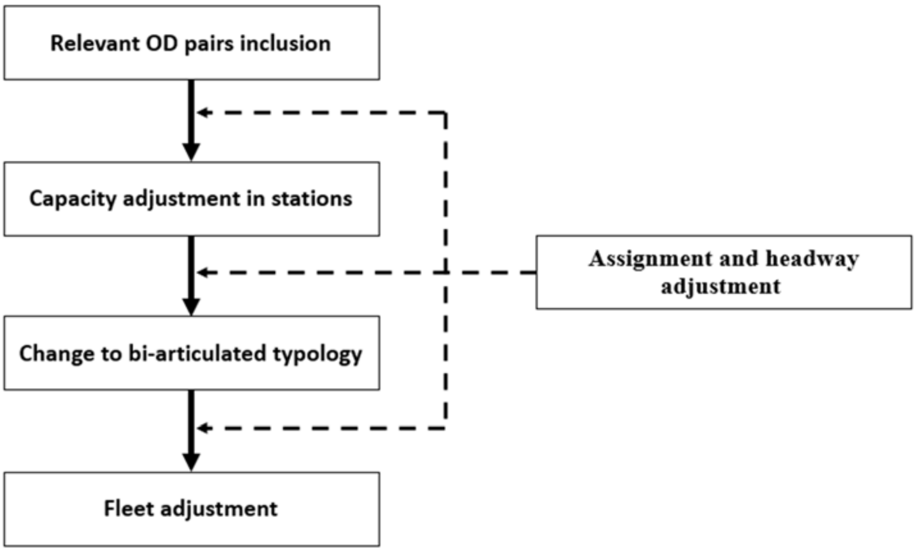


Figure 6: Selected algorithm execution order

Figure 6 shows the selected order of execution for the algorithms. Note that the assignment and headway adjustment (section 4.5) was not only performed between each of the algorithms, but also in every iteration.

6.1 Capacity adjustment in stations algorithm convergence

As stated above, the execution of CAS algorithm was completed when the percentage of saturated stations was 5% or less. This value was selected as a convergence threshold because complete compliance of station capacity restriction (0% saturated stations) could not be achieved though the continued execution of the algorithm.

Furthermore, in the scenarios when CAS was executed beyond reaching 5% of saturated stations, it continued to eliminate route stops increasing the number of transfers therefore deteriorating the overall route design.

7. RESULTS

CAS and FA algorithms were designed to achieve target values in saturated stations and bus fleets. The two are effective in this matter. A negative effect in the use of these algorithms is the increase of transfers at one or more stations, usually caused by the elimination of a route or a route stop that was the only direct alternative for a particular group of users.

To reduce transfers in a particular station, relevant origins and destinations were identified for the users performing such transfers. A stop or route that eliminated the need for transfers was then added to the preliminary design, and the algorithm was again executed. If necessary, some routes were protected from deletion as described in section 4.6.

This process was repeated iteratively for all stations with more than 4000 transfers/hour generating several scenarios until a stable point was reached. Consequently, this section presents the results for scenario 744, which satisfied system restrictions and had reasonable amounts of transfers at most stations.

CAS algorithm ran from iteration 1 to 15, and stopped when the proportion of saturated stations was under 5%. As shown in figure 7, this percentage was relatively stable for the remaining of the process, never exceeding 10%.

After CAS and CBT were completed, CAT algorithm was initiated at iteration 16. Figure 8 shows a drop in the required number of bi-articulated buses during CAT, and figure 9 shows an increase in the required number of articulated buses. CAT ended at iteration 28 when the target value was met.

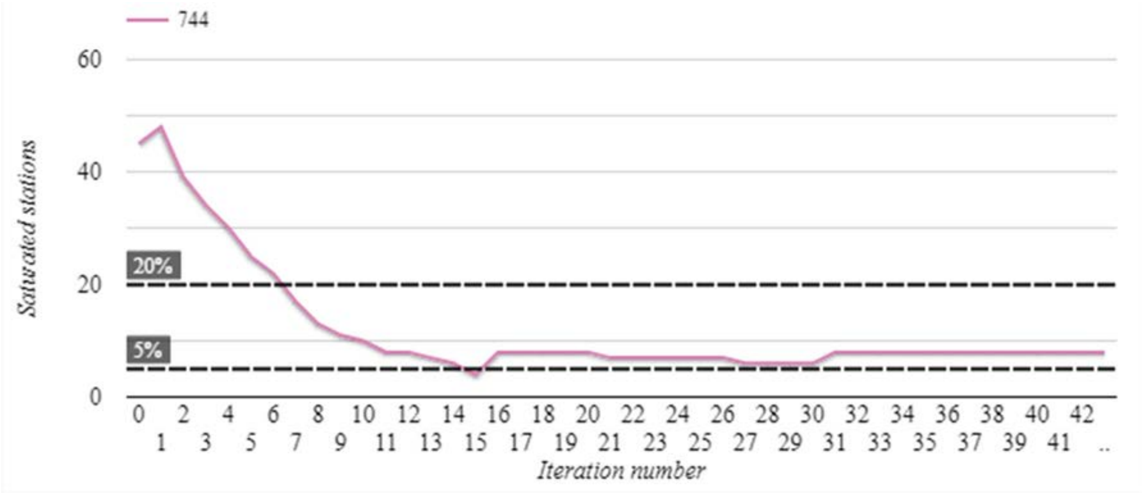


Figure 7: Percentage of saturated stations throughout algorithm execution, scenario 744

Figure 8: Required number of articulated buses throughout algorithm execution, scenario 744.

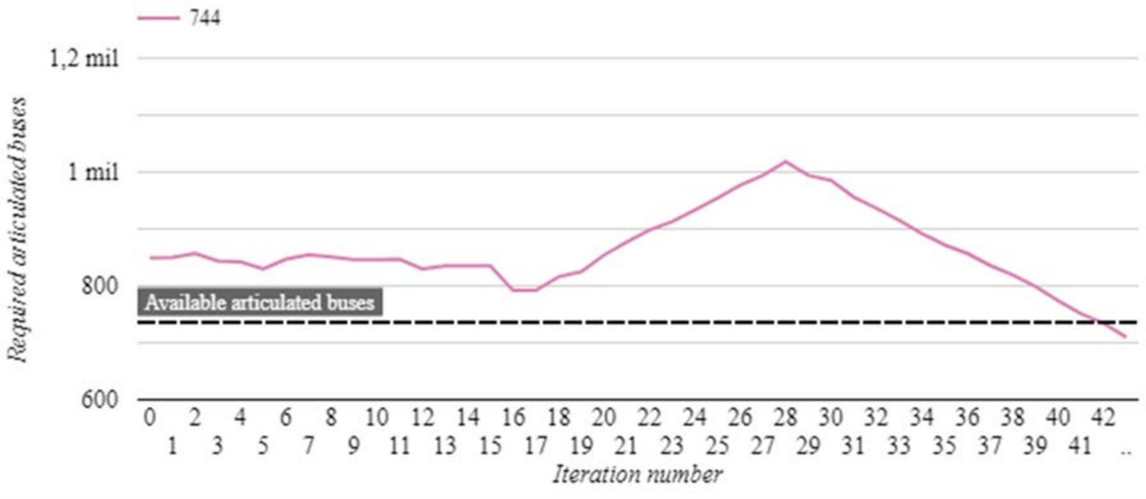


Figure 9: Required number of articulated buses throughout algorithm execution, scenario 744.

Figure 9 shows that the required number of articulated buses decreased as RE algorithm ran from iterations 28 to 43. The required number of bi-articulated buses and the percentage of saturated stations remained under acceptable ranges.

For the other indicators described in section 5, their value improved **between** base scenario and the preliminary design, but there were no significant variations when the algorithm was executed.

8. CONCLUSIONS

Process automation aids to create an operational route design proposal for a BRT system by considering a broader set of restrictions when compared with traditional methodologies.

It is possible to develop algorithms that adjust an initial route proposal for a BRT system to existing restrictions of station capacity and available bus fleet. However, the developed algorithms have an impact on system direct connectivity. In this case, planners had to manually resolve high transfer rates at some stations.

The execution of the described algorithms was not sufficient to achieve a fully applicable operational design. In the case studied, the attained percentage of saturated stations was 8%, meaning that 12 stations were to be analyzed manually to avoid sending more buses than their capacity.

Based on the iterations carried out with the CAS algorithm, it was reasonable to infer that a convergence scenario with 0% of saturated stations was not reliable, regarding the current conditions of the system. Since the saturation values in stations did not exceed the threshold of 5% in the iterations process, regardless of the elimination of service stops. Thus, in this research, it was necessary to manually restrict the algorithm to avoid undesired exclusion of several routes per station that can bring detrimental effects (stations or OD pairs not attended) so it must stop once the threshold is reached.

Despite the existence of international methodologies to generate an operational design for BRT systems, these methodologies are not definitive and do not include specific guidelines for addressing the station capacity and available fleet restrictions. The planning process for a BRT system is still open for improvement and debate.

Vehicle capacity, parallel lines, and overlapping routes can affect the results of transit assignment. Using an assignment model that takes these factors into account could improve the assignment results, and therefore the overall operational design.

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